Prestressed Stainless Steel Stayed Columns with Two Crossarms

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Received 03 May 2018; received in revised form 09 July 2018; accepted 11 August 2018

Abstract

The efficiency of prestressed stayed elements when designing very slender steel columns was proved for both stability and strength capacity of the columns. With an increasing number of crossarms placed along the length of the column the effectiveness is further growing in comparison to a stayed column with just one crossarm. First the stability of an “ideal” (perfect) prestressed stayed column with two crossarms is investigated analytically. Similar principal behavior as in the case of prestressed stayed columns with just one crossarm is confirmed and the three zones depending on the value of prestressing are revealed. Expressions for minimal, optimal and maximal prestress are derived analytically. After receiving critical buckling value of the stayed column with the two crossarms without any prestressing by linear buckling analysis (LBA), the maximal critical buckling loading of the column under optimal prestressing is derived. The results are fully demonstrated on a practical example. Second the strength capacity of such column but with relevant initial deflections is investigated by the geometrically and materially nonlinear analysis with imperfections (GMNIA) using ANSYS software. Comparisons of critical and strength values under various prestressing are analyzed with respect to a practical design. Finally some recommendations for following studies and practical use are given.

Keywords: stayed columns, two crossarms, stainless steel, prestressing, nonlinear buckling

1. Introduction

Extremely slender columns suffer with a low strength capacity due to buckling. A smart solution of the problem provide prestressed stayed steel elements. The crossarms connected by prestressed cables or rod stays with the column ends rapidly increase both the critical column load and its collapse capacity, see Fig. 1.

![Examples of structures using stayed columns](a) Grande Arche, Paris (b) Parc del centre del Poblenou, Barcelona (c) Estádio Algarve, Faro

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The prestressed stayed columns with just one crossarm were deeply investigated analytically, numerically and experimentally within several last decades. Among others, the milestones were achieved by Smith et al. [1] and Hafez et al. [2], who discovered the three zones of behavior depending on the prestressing level of the stays and Wadee et al. [3], who investigated the maximal load capacity. These results are roughly demonstrated in Fig. 2, where the “zones” may be explained as follows: zone 1 (up to $T_{min}$), where the prestressing in the stays disappears when the applied load is less or equal to the Euler load ($N_{cr} = N_E$); zone 2 (up to an optimal prestressing $T_{opt}$), where the stays remain effective until the applied load triggers a buckling; zone 3 (above $T_{opt}$), where all the stays remain active (in tension) even after the buckling. Higher prestressing than $T_{opt}$ increases the column loading and, therefore, decreases the critical column load $N_{cr}$.

Fig. 2 Stayed columns with one central crossarm

Deep investigations covering critical values, initial imperfections or maximal capacity of the stayed columns with just one crossarm were provided by Wong and Temple [4], Chan et al. [5], Saito and Wadee [6], [7], experimental and numerical investigations by Araujo et al. [8], Servitova and Machacek [9], Lima et al. [10], Osofero et al. [11], Ribeiro et al. [12], Serra et al. [12] and were mostly commented also by the Authors in [14], [15], [16], [17].

Fig. 3 Stayed columns with two crossarms

Nevertheless, both researchers and designers were aware of a greater capacity of the prestressed stayed columns with more than one crossarm since the origin of the studies (see several masts designed by Vojevodin [18], Fig. 3). While this knowledge was supported by numerical analysis and some tests (e.g. Khosla [19], Jamah and Williams [20]), the more deep investigation was performed in the last years only. Martins et al. [21] tested double crossarm stayed tube columns with a total length of 18 m, two different column cross-section diameters and various prestressing. The tests provided valuable experimental data for axial shortening and lateral deflections under loading. Yu and Wadee [22] investigated numerically triple crossarm stayed columns using ABAQUS software. Apart from varying the entry data concerning ratio of the column length to crossarms, diameter of cable stays and value of prestressing, the “efficiency indicators” were used to optimize the total design.
and optimal prestressing. For the same columns these authors [23] further developed a nonlinear analytical model verified by ABAQUS nonlinear analysis, studied parametrically buckling modes and drew attention to the possibility of a mode jumping. Lapira et al. [24] analyzed triple crossarm prestressed stayed columns and also columns with additional stay system located in the middle quarters of the system. They derived analytical formulas in 3D for maximal critical values, corresponding optimal and any other prestressing, which verified by FEM using ABAQUS software.

In this paper the Authors follow-up the mentioned previous investigations concerning stayed columns with just one crossarm and analyze 3D prestressed stainless steel stayed columns with two crossarms in accordance with Fig. 3, both analytically for “ideal” (perfect) column and numerically for “ideal” or imperfect column using ANSYS software.

2. Stability Behavior of an “Ideal” Prestressed Stayed Columns with Two Crossarms

The three approaches leading to the determination of critical loads are presented. First the geometrical analysis based on a geometry and equilibrium conditions, second linear buckling analysis (which however, as shown later, can be used for unprestressed columns only) and third the finite element method (FEM) using geometrically nonlinear analysis with imperfections (GNIA).

2.1. Geometrical analysis

First the “ideal” (perfect) column is analyzed in the similar way as was done for just one single crossarm by Smith et al. [1] and Hafez et al. [2]. The geometrical analysis of a stayed column in accordance with Fig. 3 is based on a change of element lengths due to axial deformation at the instant of buckling. The fundamental assumptions of the derivation are:

- The column is perfectly straight and concentrically loaded.
- The connections between column and crossarms are assumed to be rigid and between the stays and column/crossarms are assumed as ideal hinges.
- The maximal buckling load of the central column is assumed to be obtained by linear buckling analysis (LBA) for the column without any prestressing. This analysis assumes the stays active both in tension and compression, while neglecting buckling of stays.
- The axial deformations of the column and crossarms are not considered in LBA, however, need to be considered for analytical derivation of the magnitude of tension in the stays.

The changes of element lengths at the instant of buckling are shown in Fig. 4. The shortening of the outside and central stays gives:

\[
\Delta_c = \frac{A_c}{2} \cos \alpha - (\frac{A_c}{6} \tan \alpha) \sin \alpha - \frac{A_c}{6 \cos \alpha} = \frac{1}{3} A_c \cos \alpha - A_{vd} \sin \alpha \quad \text{and} \quad \Delta_{vm} = \frac{A_c}{3}
\]

Fig. 4 Axial compression changes of the column and force resolution in the crossarm
The spatial column has 4 stays \((n = 4)\), the planar one 2 stays \((n = 2)\). The initial axial force in the column, \(N_i\), induced by the initial stay pretension, \(T_i\), and final one, \(N_f\), after the application of the external load, \(N_a\), are:

\[
N_i = n T_i \cos \alpha \quad \text{and} \quad N_f = N_a + n T_f \cos \alpha = N_a + 4 T_f \cos \alpha
\]  

(2)

The stiffness of the column, \(K_c\), of the crossarm member, \(K_{ca}\), and of the stay, \(K_s\), are:

\[
K_c = \frac{A_c E_c}{L} \quad ; \quad K_{ca} = \frac{A_{ca} E_{ca}}{l_{ca}} \quad ; \quad K_s = \frac{A_s E_s}{l_s}
\]  

(3)

After evaluation of the shortening of the column, elongation of the crossarm member and shortening of the stay, the decrease of the tension in the stays and substitution of these values in Eq. (1) gives:

\[
T_f - T_i = \frac{N_a \cos \alpha}{3 K_c} \left( \frac{1}{K_f} + \frac{n \cos^2 \alpha + \sin^2 \alpha}{3K_c + \frac{K_{ca}}{K_s}} \right) = N_a C_{1,n}
\]  

(4)

After some substitutions the final tension in the stays, \(N_f\), may be written as:

\[
T_f = T_i - \left( \frac{N_f - n T_i \cos \alpha}{3 K_c} \right) \cos \alpha \left( \frac{1}{K_s} + \frac{\sin^2 \alpha}{K_{ca}} \right)
\]  

(5)

For the applied (external) load, \(N_a\), the substitution into Eq. (2) yields:

\[
N_a = \left( N_f - n T_i \cos \alpha \right) \left[ 1 + \frac{n \cos^2 \alpha}{3 K_c \left( \frac{1}{K_s} + \frac{\sin^2 \alpha}{K_{ca}} \right)} \right] = \left( N_f - n T_i \cos \alpha \right) C_{2,n}
\]  

(6)

Using these formulas, the behavior of the stayed column with the two crossarms may be described similarly as behavior of the column with just one crossarm. In Fig. 5 the resulting three zones of behavior with respect to value of prestressing are shown. Also, for a column with two crossarms analyzed in following Paragraphs (see the entry data given in Paragraph 2.2) the buckling modes and critical loads of unprestressed column are shown.

![Fig. 5 Initial pretension vs buckling load and example of the first three buckling modes corresponding to the later analyzed column using LBA](image)

Zone 1:

The initial prestressing is small \((T_i \leq T_{min})\) and after external loading disappears. The column behaves as unstayed and buckles at the Euler load, \(N_E\). Minimal prestressing, \(T_{min}\) corresponding to this behavior, follows from Eq. (4):
\[ N_{cr} = N_c = \frac{\pi^2 E_c I_c}{L^2} \quad ; \quad T_i = T_{min} = N_a C_{1,n} = N_k C_{1,n} = \frac{\pi^2 E_k I_c}{L^2} C_{1,n} \]  

(7)

Zone 2:

The prestressing is larger than the minimal one but smaller than the optimal one, \( T_{min} < T_f < T_{opt} \). After triggering of buckling the prestressing in the stays disappears \( (T_f = 0) \), but the stays at convex side will become active immediately. The resulting critical load, \( N_{cr} \), will be higher than Euler’s one and according to Eq. (4) will correspond to prestressing \( T_i \):

\[ N_{cr} = N_a = \frac{T_i}{C_{1,n}} \]

(8)

Maximal critical load, \( N_{cr,max} \), follows from Eq. (6), after substituting for \( N_a = N_{cr,n,T=0} \) where the latter is the value of critical load received from LBA for fully active stays without any prestressing \( (T_i = 0) \), see Fig. 5:

\[ N_{cr,max} = N_f = \frac{N_{cr,n,T=0}}{C_{2,n}} \]

(9)

Corresponding optimal prestressing is given by:

\[ T_{opt} = N_{cr,max} C_{1,n} \]

(10)

Zone 3:

The prestressing is larger than the optimal one. After buckling the stays remain in a tension and the critical load falls off due to respective force components in the stays. The residual tension in the stays follows from Eqs. (5) and (9) as:

\[ T_r = T_i - \left( N_{cr,max} - n T_i \cos \alpha \right) \frac{\cos \alpha}{3 K_s \left( \frac{1}{K_s} + \frac{\sin^2 \alpha}{K_{ca}} \right)} = T_i - \left( N_{cr,max} - n T_i \cos \alpha \right) C_3 \]

(11)

The critical load in Zone 3 can be obtained from Eq. (6) after substituting for \( N_a = N_c \) and \( N_f = N_{cr,max} \):

\[ N_{cr} = \left( N_{cr,max} - n T_i \cos \alpha \right) C_{2,n} \]

(12)

2.2. Numerical stability analysis using 3D GNIA

The analytical investigation of the critical load based on geometrical analysis of an “ideal” (perfect) central column may be verified by using FEM. In the field of prestressed elements the stability behavior in the full range of prestressing can’t be solved by linear buckling analysis (LBA) but due to the sudden change of inner energy at the instant of buckling the geometrically nonlinear analysis with imperfections (GNIA) is necessary (see, Saito and Wadee [5] or previous articles of the Authors, e.g. [15], [17]). Under a small prestressing the stay at buckling become slacked and the ones at convex side are immediately activated (Zones 1 and 2). If all the stays, both at convex and concave sides are activated, the column behavior corresponds to the Zone 3.

The actual behavior of the stayed column with the two crossarms was therefore investigated by using ANSYS software in 3D and geometrically nonlinear analysis with imperfections (GNIA). The results are demonstrated on the same example as analyzed by the Authors in [16], however, now with the two crossarms instead of one crossarm. The entry data (length, area, second moment of area, modulus of elasticity) are as follows, see Fig. 3:

- Central stainless steel tube \( \varnothing 50 \times 2 \) [mm]: \( L = 5000 \) mm, \( A_c = 302 \) mm2, \( I_c = 87009 \) mm4, \( E_c = 200 \) GPa.
- Crossarm stainless steel tubes \( \varnothing 25 \times 1.5 \) [mm]: \( a = 250 \) mm, \( A_a = 111 \) mm2, \( I_a = 7676 \) mm4, \( E_a = 200 \) GPa.
- Stays as Macalloy cables 1x19 stainless steel Ø 4 mm: $L_s = 2513$ mm, $A_s = 12.6$ mm², $E_s = 200$ GPa.

Substituting these values into the formulas presented in Chapter 2.1 yields:

- The invariables: $C_{1,4} = 0.0351$, $C_{2,4} = 1.1612$.

Critical loads and prestressing (using $N_{cr,1,T=0} = 44.43$ kN from LBA, see Fig. 5):

- $N_E = 6870$ N; $T_{min} = 241$ N; $N_{cr,max} = 38262$ N; $T_{opt} = 1343$ N; $N_{cr,T_{opt}} = 25925$ N.

The ANSYS model involved BEAM188 elements for the central and crossarm tubes and LINK180 elements for the cable stays, with the same boundary conditions as mentioned in the Chapter 2.1 (connections between column and crossarms are assumed to be rigid, between stays and column/crossarms are ideal hinges). The meshing study resulted into division of $L/250$ and $a/25$ as satisfactory one. First, the required initial deflections were introduced, followed by the relevant prestressing of stays through their thermal change (i.e. by cooling). Finally, axial deflections to the central column (giving an external load) up to collapse were imposed. A standard Newton-Raphson iteration was used. To verify the analytical values, first GNIA was used, with elastic material behavior, followed by GMNIA for stainless steel material.

For the stability analysis the “ideal” column was analyzed, the initial imperfections need to be negligible, therefore infinitesimal deflections were employed with the amplitude of $w_0 = L/500000 = 0.01$ mm (in the 3D as $w_{0y} = w_{0z} = w_0/\sqrt{2}$) for both symmetric and antisymmetric modes, see Fig. 6.

![Fig. 6 Initial deflections](image)

The resulting load-prestressing relationship received from the GNIA is presented in Fig. 6. The curves for critical and maximal loading were received from rather lengthy numerical calculations of 26 prestressing values (in reality cooling of the stays), each with 1000 of compression steps. In low prestressing, roughly up to 1.6 kN, when at the instant of attaining the critical load (i.e. when the tension in all stays become zero) the both stays at the convex side become active in tension (while at concave side being slacked) and the maximal (capacity) load of the “ideal” column becomes higher than the critical one. With higher value of prestressing the all four stays remain in some tension (Zone 3) and both critical and maximal values are identical (show also the explanation in Fig. 8 for GMNIA).

Comparison of analytical and numerical (GNIA) values is rather difficult (see Fig. 6) but the analytical values, i.e. $N_{cr,analyt} = 38262$ N at prestressing of $T_{opt,analyt} = 1343$ N, may be compared with the GNIA ones when the stays on the concave side at the buckling don’t slacken and critical and maximal loads merge together, i.e. $N_{cr,GNIA} = 38200$ N at prestressing of $T_{opt,GNIA} = 1599$ N, being acceptable. Nevertheless, GNIA maximal load for “ideal” column of roughly 52618 N arises at prestressing of 9808 N. It should be noted, that deflected shape for both modes of negligible initial deflection are identical, after $N_{cr} = 47852$ N first interactive (in combination of symmetric and antisymmetric mode), after prestressing of $T = 9808$ N becomes antisymmetrical.
3. Maximal Loading of an Imperfect Prestressed Stayed Columns with Two Crossarms

The investigation of a maximal loading (strength capacity) of a compression element requires the modelling of a real, imperfect column. The initial imperfections in such stayed columns were introduced in accordance with Eurocode EN 1993-1-1 as equivalent initial deflections with amplitude \( w_0 = L/200 = 25 \text{ mm} \) (valid for cold-formed thin-walled tubes and elastic analysis). The deflection was considered again in the spatial direction, i.e. with \( w_0 = w_0 \sqrt{2} \). The first two modes of initial deflections were considered only, in accordance with figure 5: antisymmetrical with the amplitude \( w_0, L/2 = L/400 \) and symmetrical with the amplitude \( w_0 = L/200 \).

The GNIA for the above analyzed example with the same stainless steel elastic modulus (according to EN 1993-1-4) \( E = 200 \text{ GPa} \) resulted in various prestressing into values presented in Fig. 7. As in the LBA the decisive mode proved to be the antisymmetric one, giving maximal loading \( N_{\text{max}} = 31988 \text{ N} \) at prestressing of \( T_i = 8353 \text{ N} \).

Another study employed GMNIA (geometrically and materially nonlinear analysis with imperfections), differing just in using the nonlinear material behavior as tested by the Authors in [16], resulting in the stress-strain relationship acc. to Fig. 8.
Again both the “ideal” (perfect) central columns with infinitesimal amplitude of initial deflections \( w_0 = L/500000 = 0.01 \, \text{mm} \) were analyzed (an example of one analysis under initial prestressing of \( T_i = 1278 \, \text{N} \) is shown for illustration in Fig. 8). The results for the critical loads (concerning “ideal” stayed columns) for antisymmetrical initial deflections now partly differ from results in the same column but with symmetrical initial deflections, show in Fig. 9. Nevertheless, the important value of \( N_{cr,GMNIA} = 34429 \, \text{N} \) at prestressing of \( T_{opt,GMNIA} = 1583 \, \text{N} \) corresponds well with the former GNIA \( N_{cr,GNIA} = 38000 \, \text{N} \) at prestressing \( T_{opt,GNIA} = 1599 \, \text{N} \), considering the ratio of the elastic moduli in GNIA/GMNIA as \( E_{\text{in}}/E_{\text{opt}} = 184/210 = 0.92 \). A simple reduction using this factor gives \( N_{cr,GNIA,E=184 \, \text{MPa}} = 34960 \, \text{N} \) which differ from \( N_{cr,GMNIA} = 34429 \, \text{N} \) by 1.5 %.

The GMNIA results of the real column with an amplitude of initial deflection of \( w_0 = L/200 = 25 \, \text{mm} \) are shown in Fig. 10. In comparison with GNIA (Fig. 7) the maximal (capacity) loadings are significantly lower due to nonlinear stress-strain relationship of the stainless steel material. The decrease of the maximal loading for decisive antisymmetrical initial deflection mode from 31988 \, \text{N} \) to 25040 \, \text{N} \) gives 27.7 %, while for symmetric initial deflection mode the drop from 41544 \, \text{N} \) to 34000 \, \text{N} \) is 22.2 %.

4. Conclusions

The paper investigates prestressed stainless steel stayed columns with two crossarms, located at the thirds of the central column length. In the first part the analytical stability of an “ideal” column is investigated. Based on 2D geometrical analysis the behavior of the column is described for an arbitrary prestressing and general geometry. Similarly as in the case of the stayed columns with just one central crossarm the three zones according to the extent of prestressing are revealed and formulas for minimal and maximal critical loadings in 2D and 3D together with “optimal” prestressing are derived.

The resulting formulas are applied to a practical example, which has all the geometric and material characteristics the same as for the stayed column which was investigated both experimentally and theoretically by the Authors in the past, presented e.g. in [17]. The only difference consists in the two crossarms in the thirds of the column length instead of just one central crossarm.
Numerical analysis started with a linear buckling analysis (LBA) under zero prestressing and stays active in compression, giving critical loadings and deflection modes required for the use in the above analytical solution. Subsequently a 3D model in ANSYS software was prepared to analyze both the stability of “ideal” (perfect) column and maximal loading (column capacity) of the “real” (imperfect) stayed column. The “no compression option” for the stays was adopted to simulate any slackening of these elements and respective initial deflections were introduced: infinitesimal ones for critical loadings in the case of “ideal” column \((L/500000)\) and required ones according to Eurocode 1993-1-1 for imperfect column \((L/200\) for thin-walled cold-formed members). The initial deflection modes were introduced in accord with LBA either as antisymmetrical one with two half sine waves or symmetrical one with the full sine wave.

The main results of the investigation may be summarized as follows:

(a) The analytical formulas for the double crossarm prestressed stayed columns concerning minimal and optimal prestressing giving maximal critical analytical loading were derived.

(b) Numerical modelling (using ANSYS software) of the column with a practical size was presented for a verification of the analytical approach. It was shown, that the stability behavior of an “ideal” prestressed column is more complex in comparison to the simple analytical approach and a postcritical behavior even for “ideal” column need to be evaluated. Nevertheless, the analytical value of the maximal critical loading was well comparable with the numerical one for the prestressing when the stays on the concave side at buckling do not slacken.

(c) Numerical analysis using GNIA with the initial deflections of the main column corresponding to thin-walled cold-formed tubes (with amplitude \(L/200\)) gives significantly lower maximal (capacity) loading in comparison with the critical loading. This decrease in the comparison with analytical critical loading in the specified shown example is at 84\% and in comparison to maximal loading of “ideal” column is roughly at 61\%.

(d) Considering GMNIA with the stainless steel material nonlinearity leads to even greater reduction in comparison to the former values. The decrease of maximal (capacity) loading due to initial lower Young’s modulus and due to material nonlinearity is another 27.7\%.

(e) Finally the comparison of effectivity between the stayed column with just one crossarm (see [17]) and the column with the two cross arms (with the above specified geometry otherwise unchanged) in the GNIA shows due to adding the second crossarm increase of both critical loading of “ideal” column and maximal loading of imperfect column. The increase of the critical loading is 21.0\% (from 31580 N to 38200 N) and increase of the maximal loading is 63.5\% (from 19570 N to 31988 N).

Although the presented percentage values are perfectly valid just for the specific geometry only, the higher efficiency of the two crossarms is undisputable and may provide designers with a guidance for more economical and reliable design.

**Acknowledgement**

Support of the Czech Grant Agency grant GACR No. 17-24769S is gratefully acknowledged.

**References**


