

# **Long Pinions for Alternative Transmission Mechanizations**

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## **Abstract**

Multi-speed transmissions are used in the automotive industry. The design of such transmissions may be framed as an algebraic/graph theory problem. Such an approach is particularly useful for designs with multiple planetary gear sets. Some automotive manufacturers use “long pinions” or “shared pinions” as building block elements within these gear sets. The long pinion is an interesting architectural arrangement that combines certain packaging and functional attributes. In the present article, the algebraic use of the long pinion is demonstrated in the creation of vastly different transmission architectures from the same skeleton diagram.

**Keywords:** automotive transmissions, shared pinion, long pinion

## **1. Introduction**

The 2016 International Energy Outlook [1], offers the following predictions. Global population grows 23% from 7.3 billion in 2015 to 9.0 billion in 2040. Total world marketed energy grows by 42% from 575 quads in 2015 to 815 quads in 2040 (quad = quadrillion Btu). Fossil fuels still supply over 75% of world energy in 2040. Petroleum remains the world’s largest energy source and worldwide energy-related CO<sub>2</sub> emissions rise from 34 billion metric tons per year in 2015 to 43 billion metric tons per year in 2040, a 29% increase. Against this backdrop of increased population, urbanization, and increased energy usage and emissions, we continue to explore novel automotive propulsion architectures with improved efficiency. The problem of designing ultra-compact transmissions is a valuable aspect of this research as it frees up room for the insertion of electrification components within the packaging envelopes of existing propulsion systems.

A vehicle transmission delivers mechanical power from the engine to the remainder of the drive system, such as fixed final drive gearing, axles and wheels. The mechanical transmission enables some freedom in engine operation, usually through alternate selection of multiple drive ratios, a neutral selection that allows the engine to operate accessories with the vehicle stationary, and clutches and a torque converter for smooth transitions between driving ratios and to start the vehicle from rest with the engine turning. An electrically variable transmission (EVT) is a mechanical transmission augmented by one or more electric motor/generators. An EVT uses differential gearing to send a fraction of its transmitted power through an electric path to the final drive. The remainder of its power flows through another, parallel path that is all mechanical and direct, of fixed ratio, or alternatively selectable.

## **2. Background and Prior Work**

The theory of multi-speed transmission kinematic operation has been traditionally handled via lever diagrams [2]. These are convenient graphical representations of transmission gear ratios in terms of the relative dimensions of ring, sun, and planetary gear sets. Since the automotive industry is generally moving in the direction of larger numbers of fixed speed ratios

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as well as hybrids, we briefly review recent work on multi-speed transmissions and EVT's. Lewis and Bollwahn [3] present the General Motors Hydra-Matic/Ford six-speed FWD automatic transmission family. Designed-in modularity requires only changes to the second and third axis and case housings to achieve various torque requirements as stipulated by the specific vehicle application. Singh and Olenzek [4], discuss variants of General Motors' family of small front wheel drive six speed automatic transmissions including the 6T40 and 6T45, which cover a range of vehicles from small and compact cars to small SUVs and handle engines torque capacities up to 240 N-m Gas (280 N-m Diesel) & 315 N-m Gas (380 N-m Diesel), respectively. Clark et al., [5], describe General Motors' front wheel drive seven speed dry dual clutch automatic transmission introduced in 2014. The 250 N-m input torque rated gear box was designed and engineered for a global market in both front wheel drive and all-wheel drive configurations. Raghavan [6], introduces the eight speed rear wheel drive transmission family recently launched by General Motors. This set of designs yields a 7.0 overall ratio spread, enabling improved launch capability because of a deeper first gear ratio and better fuel economy due to lower top gear N/V capability. Grewe et al., [7], describe the GM Two-Mode Hybrid transmission for full-size, full-utility SUVs. This system integrates two electromechanical power-split operating modes with four fixed gear ratios and provides fuel savings from electric assist, regenerative braking and low-speed electric vehicle operation. Miller et al., [8] describe the Voltec 4ET50 multi-mode electric transaxle, which introduces a unique two-motor EV driving mode that allows both the driving motor and the generator to simultaneously provide tractive effort while reducing electric motor speeds and the total associated electric motor losses.

In the present paper, we show how such planetary gear set based designs could potentially be made more compact via the use of long pinions. This work is a natural extension of the results first presented by Raghavan [9].

### 3. Lever Diagrams and Planetary Geartrains

The primary building block of a planetary gear train is the planetary gear set shown in Fig. 1. It is comprised of a sun gear, a ring gear, and a set of planet gears (also known as pinion gears). Fig. 1(a) shows a "simple" planetary gear set and Fig. 1(b) shows a "double-pinion" planetary gear set. The planet gears are carried on a carrier member. The entire system of Fig. 1(a) or 1(b) may be represented by an edge-vertex graph (a vertical straight line segment with 3 nodes on it) with the nodes labeled as ring R, carrier C, and sun S, respectively (see Fig. 1(c)). Such a graph is also known as a "lever," because the relative rotational speeds of ring, carrier, and sun may be computed by treating rotational speeds as forces acting on the lever, and taking moments about appropriate nodes on the lever [2].

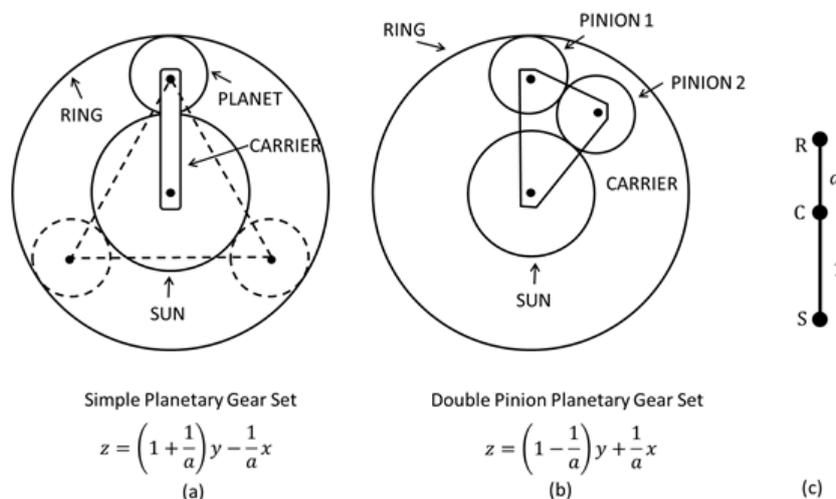


Fig. 1 Simple planetary and double pinion planetary gear sets

An alternative to this geometric representation of levers is the following algebraic representation. Let  $x$ ,  $y$ , and  $z$  be the speeds of the ring, the carrier, and the sun in Fig. 1(a). Then these speeds are related by the following equation:

$$z = \left(1 + \frac{1}{a}\right)y - \frac{1}{a}x \quad (1)$$

This relationship holds true by virtue of the mechanical interconnections and gear interactions in the planetary gear set (see Shigley and Uicker [10]). Eq. (1) contains a parameter  $a$ , which is equal to the ratio  $\frac{n_S}{n_R}$ ,  $n_S$  and  $n_R$  being the numbers of teeth on the sun gear and the ring gear, respectively. For double-pinion planetary gear sets (see Fig. 1(b)) the governing equation is:  $z = \left(1 - \frac{1}{a}\right)y + \frac{1}{a}x$ .

#### 4. Long Pinion Arrangements

Given 2 adjacent planetary gear sets, we say that they use a “long pinion” if they share a common carrier and an integral pinion member. We can write expressions for the long-pinion arrangement as follows.

Case 1: For a combination of a simple planetary gear set (henceforth abbreviated as PG) connected to another simple PG via a long-pinion carrier, the constraint equations are:

$$z_1 = -k_1x_1 + (1 + k_1)y_1, \text{ (first planetary)} \quad (2)$$

$$z_2 = -k_2x_2 + (1 + k_2)y_2, \text{ (second planetary)} \quad (3)$$

$$y_2 = y_1, \text{ (common carrier)} \quad (4)$$

where  $k_i = \frac{1}{a_i}$ ,  $i=1,2$ .

Additionally, the pinion speed of the first PG,  $\Omega_1 = \frac{(x_1 - y_1)2k_1}{(k_1 - 1)}$ , is set equal to the pinion speed of the second PG,  $\Omega_2 = \frac{(x_2 - y_2)2k_2}{(k_2 - 1)}$ , because the pinion is shared between the 2 PGs. The reader is referred to the work of Shigley and Uicker [10] for more details on how to derive the kinematic equations relating the speeds of the ring, carrier, sun, and pinion gears.

$$\frac{(x_1 - y_1)2k_1}{(k_1 - 1)} = \frac{(x_2 - y_2)2k_2}{(k_2 - 1)}. \text{ (common pinion)} \quad (5)$$

By using Eqs. (4) and (5) to eliminate the variables  $x_2$  and  $y_2$  from Eq. (3) in terms of  $x_1$  and  $y_1$ , we get the following equation

$$z_2 = -\left(\frac{k_2\xi_2}{\xi_1}\right)x_1 + \left(1 + \frac{k_2\xi_2}{\xi_1}\right)y_1. \text{ (second planetary)} \quad (6)$$

where  $\xi_i = (1 - a_i)$ ,  $i=1,2$ . Eqs. (2) and (6) serve as constraints relating the rotational speeds of the 4 independent nodes  $\{x_1, y_1, z_1, z_2\}$  of the 2-planetary system comprised of PG1 and PG2 together with the common carrier and pinion.

Case 2(a): PG1 is simple and PG2 is of the double-pinion type

$$z_1 = -k_1x_1 + (1 + k_1)y_1, \text{ (first planetary)} \quad (7)$$

$$z_2 = k_2x_2 + (1 - k_2)y_2, \text{ (second planetary)} \quad (8)$$

$$y_2 = y_1. \text{ (common carrier)} \quad (9)$$

The pinion speed of the first PG is to be set equal to the pinion speed of the second PG. However, the second PG has a double pinion arrangement and so we have a choice of either Pinion 1 or Pinion 2 (see Fig. 1(b)). If we use Pinion 1, its speed

in terms of other parameters of PG2 is  $\frac{R_2}{P_2}(x_2 - y_2)$  where  $P_2$  and  $R_2$  are respectively, the pinion diameter and the ring gear diameter on PG2. We can see that for Pinion 1,  $P = \xi \frac{R-S}{2}$ , where  $\xi < 1$ . By using this result, the speed of Pinion 1 may be expressed as  $\frac{(x_2 - y_2)}{\xi(1 - a_2)}$ . Setting this equal to the speed of one of the pinions on PG1, we get

$$\frac{(x_1 - y_1)2k_1}{(k_1 - 1)} = \frac{(x_2 - y_2)}{\xi(1 - a_2)} \tag{10}$$

Upon rearrangement of terms in Eq. (10), we get

$$x_2 = \left( \zeta \frac{\xi_2}{\xi_1} \right) x_1 + \left( 1 - \zeta \frac{\xi_2}{\xi_1} \right) y_1 \tag{11}$$

Substituting this expression for  $x_2$  in Eq. (8), and using Eq. (9) to eliminate  $y_2$ , and then, re-arranging terms, we get

$$z_2 = \left( k_2 \zeta \frac{\xi_2}{\xi_1} \right) x_1 + \left( 1 - k_2 \zeta \frac{\xi_2}{\xi_1} \right) y_1 \tag{12}$$

Eqs. (7) and (12) serve as constraints relating the rotational speeds of the 4 independent nodes  $\{x_1, y_1, z_1, z_2\}$  of the 2-planetary system comprised of PG1 and PG2 together with the common carrier and common pinion fixed interconnection between the simple planetary gear set pinion and Pinion 1 on the double pinion planetary gear set. Similar expressions can be worked out for the cases listed Table 1 .

Table 1 Gearset combinations

2(b)	PG1 is simple; PG2 is double pinion	Pinion-Pinion2
3(a)	PG1 is double pinion; PG2 is simple	Pinion 1-Pinion
3(b)	PG1 is double pinion; PG2 is simple	Pinion 2-Pinion
4(a)	PG1 is double pinion; PG2 is double pinion	Pinion 1-Pinion 1
4(b)	PG1 is double pinion; PG2 is double pinion	Pinion 1-Pinion 2
4(c)	PG1 is double pinion; PG2 is double pinion	Pinion 2-Pinion 1
4(d)	PG1 is double pinion; PG2 is double pinion	Pinion 2-Pinion 2

## 5. Long Pinion Transformations

### Example

Let's take a look at a 3 planetary gear set design with 2 brakes and 3 clutches yielding 6 forward speeds and 1 reverse speed as shown in Fig. 2 along with the clutching table. In the following we use a long pinion transformation to create a more compact alternative to the design of Fig. 2. If we focus on just the first 2 planetary gearsets with their fixed interconnections, the sub-system is as shown in Fig. 3(a). The associated equations are:

$$z_1 = -k_1 x_1 + (1 + k_1) y_1, \quad (\text{first planetary}) \tag{13}$$

$$z_2 = -k_2 x_2 + (1 + k_2) y_2, \quad (\text{second planetary}) \tag{14}$$

$$x_1 = y_2, \quad (\text{fixed interconnection 1}) \tag{15}$$

$$y_1 = x_2. \quad (\text{fixed interconnection 2}) \tag{16}$$

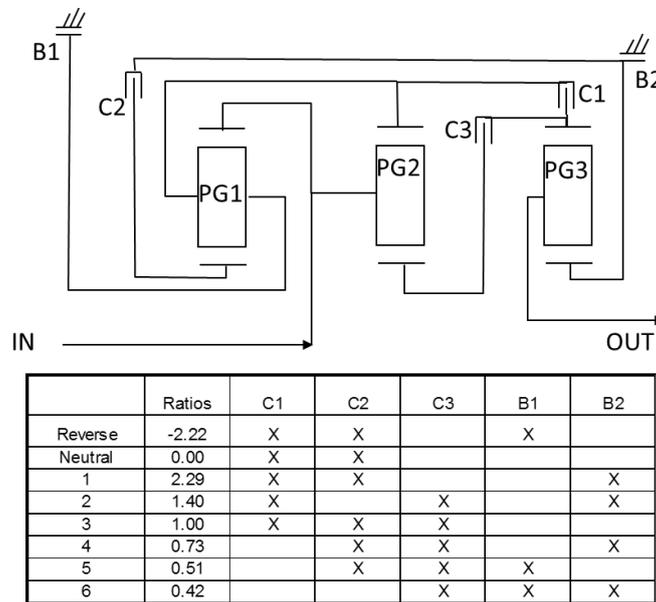
Substituting for  $y_2$  from Eq. (15) and for  $x_2$  from Eq. (16) into Eq. (14), we get

$$z_2 = (1 + k_2) x_1 + (1 - (1 + k_2)) y_1 \tag{17}$$

Eqs. (13) and (17) taken together represent the physical system shown in Fig. 3(b), featuring a simple planetary gear set coupled to a double pinion planetary gear set via fixed interconnections, Ring to Ring, and Carrier to Carrier. Going a step further we may compare Eq. (17) with Eq. (12) which represents the governing equation of a double pinion planetary gear set which shares a  $P_1$  pinion and carrier with an adjacent simple planetary gear set. Equating corresponding coefficients, we get

$$1 + k_{2\_original} = k_{2\_LP} \zeta_{LP} \frac{\xi_{2LP}}{\xi_{1LP}} \tag{18}$$

where the subscripts *original* and *LP* apply respectively to the original gear sets of Fig 3(a) and the long pinion gear sets that we wish to transform them into. Using the numerical values for the  $\frac{Ring}{Sun}$  ratios from Fig. 3(a), we set  $k_{1\_original} = 1.57, k_{2\_original} = 2.37, \zeta_{LP} = 0.9$ . Further, setting  $k_{1\_LP} = k_{1\_original}$ , we may compute  $k_{2\_LP}$  from Eq. (18). Its value is 2.35. By this process, the arrangement in Fig. 3(b) with Ring-Ring and Carrier-Carrier fixed interconnections, transforms into the long pinion arrangement of Fig. 3(c), with  $\left(\frac{Ring}{Sun}\right)_1 = 1.57, \left(\frac{Ring}{Sun}\right)_2 = 2.35$ .



(X = engaged clutch)

$$\frac{R_1}{S_1} = 1.57, \frac{R_2}{S_2} = 2.37, \frac{R_3}{S_3} = 2.50$$

Fig. 2 Three planetary six speed transmission

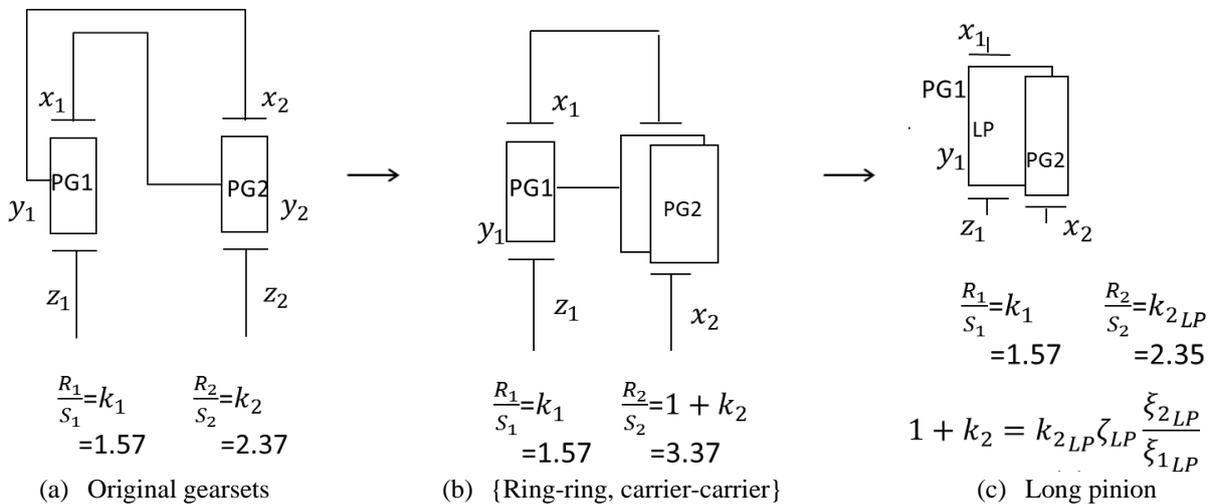


Fig. 3 Transformation to long pinion

Kinematically, the 3 arrangements of Fig. 3 are equivalent. Therefore, we may substitute the arrangement of Fig. 3(c) in place of the first 2 planetary gearsets in Fig. 2 to get Fig. 4. Note that the speed ratios and clutch sequences are identical for Figs. 2 and 4. The arrangement of Fig. 4 can be implemented with a shorter axial length relative to Fig. 2 due to the integration of the first 2 PGs via the long pinion transformation.

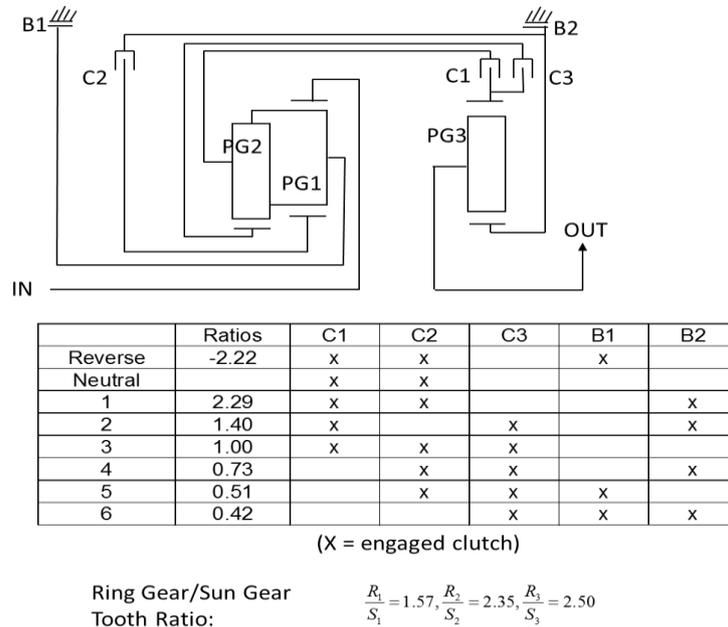


Fig. 4 Six speed transmission with long pinion

## 6. Conclusions

In the preceding sections, we have demonstrated the algebraic use of long pinion arrangements in the creation of alternative architectures for automotive transmissions. Once a powerflow with favorable characteristics is established, there are many ways in which it can be mechanized. We have shown the use of long pinion arrangements to replace (Ring-Carrier, Carrier-Ring) connections in adjacent planetary gear sets, while retaining functional equivalence. Various other permutations and combinations of fixed interconnections may be worked out to achieve ultra-compact designs.

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