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Advanced Manufacture of Spiral Bevel and Hypoid Gears

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Abstract

In this study, an advanced method for the manufacture of spiral bevel and hypoid gears on CNC hypoid generators is proposed. The optmal head-cutter geometry and machine tool settings are determined to introduce the optimal tooth surface modifications into the teeth of spiral bevel and hypoid gears. The aim of these tooth surface modifications is to simultaneously reduce the tooth contact pressure and the transmission errors, to maximize the EHD load carrying capacity of the oil film, and to minimize power losses in the oil film. The proposed advanced method for the manufacture of spiral bevel and hypoid gears is based on machine tool setting variation on the cradle-type generator conducted by optimal polynomial functions and on the use of a CNC hypoid generator. An algorithm is developed for the execution of motions on the CNC hypoid generator using the optimal relations on the cradle-type machine. Effectiveness of the method was demonstrated by using spiral bevel and hypoid gear examples. Significant improvements in the operating characteristics of the gear pairs are achieved.

Keywords: manufacture, spiral bevel and hypoid gears, load distribution, EHD lubrication, CNC generator

1. Introduction

The new CNC hypoid generators have made it possible to perform nonlinear correction motions for the cutting of the face-milled and face-hobbed spiral bevel and hypoid gears. Several studies investigated freeform cutting methods using such machines. Among them, Shih and Fong [1] proposed a flank-correction methodology derived directly from the six-axis Cartesian-type CNC hypoid generator. A polynomial representation of the universal motions of machine tool settings on CNC machines was proposed by Fan in Ref. [2]. Chen and Wasif [3] presented a new mathematical model to calculate the cutter system location and orientation and a generic post-processing method to establish the machine kinematic chain and to compute the coordinates of the machine axes for the face-milling process on CNC machines. Zhang et al. [4] derived the relative motion relation among the virtual cradle, generating gear, cutter and workpiece on the CNC hypoid generator.

To achieve maximum life in a gear set, appropriate bearing pattern location with low tooth contact pressure and low loaded transmission error must coexists. The maximum tooth contact pressure and transmission error depend substantially on tooth geometry. In order to reduce the tooth contact pressure and the transmission errors, and to decrease the sensitivity of the gear pair to errors in tooth surfaces and to the relative positions of the mating members, carefully chosen tooth surface modifications are usually applied to the teeth of one or both mating gears. As a result of these modifications, a point contact replaces the theoretical line contact of the fully conjugated tooth surfaces. These modifications are introduced into the gear tooth surfaces by applying the appropriate machine tool setting for the manufacture of the pinion and the gear and/or by using a head-cutter with optimized geometry. The new CNC hypoid generators have made it possible to perform varying correction motions during the cutting of face-milled and face-hobbed spiral bevel and hypoid gears. In this paper, a method is presented to determine optimal head-cutter geometry and optimal polynomial functions for the conduction of machine tool setting variation in pinion teeth finishing simultaneously reducing maximum tooth contact pressure and transmission errors, maximizing EHD load carrying capacity of the oil film, and minimizing the power losses in the oil film. The developed optimization procedure relies heavily on the loaded tooth contact analysis for the prediction of maximum tooth contact pressure and trans-

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mission errors and on the elastohydrodynamic lubrication analysis for the calculation of EHD load carrying capacity of the oil film, and power losses in the oil film. The load distribution and transmission error calculation method employed in this study was developed by the author of this paper [5, 6]. The EHD lubrication calculations are based on the method presented in Refs. [7, 8] The optimization is based on machine tool setting variation on the cradle-type generator conducted by optimal polynomial functions and on optimal head-cutter geometry. In the second step an algorithm is developed for the execution of motions on the CNC hypoid generator using the relations on the cradle-type machine. Effectiveness of the method was demonstrated by using spiral bevel and hypoid gear examples. Significant reductions in the maximum tooth contact pressure and transmission errors, and improvements in lubrication performances were obtained.

2. Manufacture of Spiral Bevel and Hypoid Gears on Cradle-Type Generator

The concept of an imaginary generating crown gear is used in the generating cutting process of the face-hobbed spiral bevel and hypoid pinion and gear teeth (Fig. 1). The machine tool settings are: The tilt angle of the cutter spindle with respect to the cradle rotation axis (κ), the swivel angle of cutter tilt (μ), the radial machine tool setting (e), and the tilt distance from tilt centre to reference plane of head-cutter (h_d). To obtain the tooth surface in the generating process, the work gears are rolled with the imaginary generating gear. The coordinate systems $K_e(x_e, y_e, z_e)$ and $K_i(x_i, y_i, z_i)$ are attached to the head-cutter and to the pinion/gear, respectively. The teeth-surfaces of the pinion and of the gear are defined by the following system of Eqs. (9)-(10):

$$\vec{\mathbf{r}}_i^{(i)} = \mathbf{M}_{i3} \cdot \mathbf{M}_{i2} \cdot \mathbf{M}_{i1} \cdot \mathbf{M}_{c4} \cdot \mathbf{M}_{c3}\left(e, i_{g1}\right) \cdot \mathbf{M}_{c2}\left(\mu, \kappa, h_d\right) \cdot \mathbf{M}_{c1}\left(r_{t0}\right) \cdot \vec{\mathbf{r}}_e^{(i)} \quad (1)$$

$$\vec{\mathbf{v}}_{c0}^{(i,c)} \cdot \vec{\mathbf{e}}_{c0}^{(i)} = 0$$

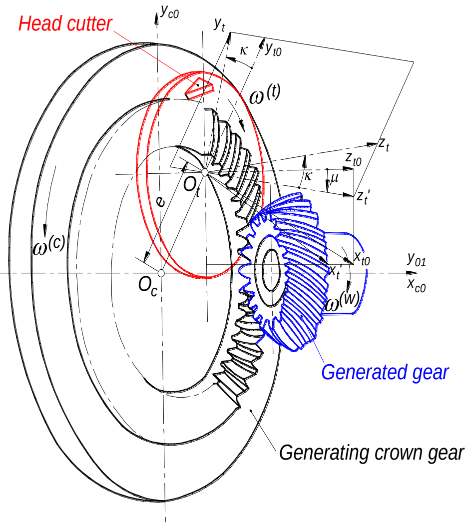

Fig. 1 Spiral bevel gear hobbing

2.1. Variation of Machine Tool Setting Parameters

The variations of the tilt and swivel angles, tilt distance, radial machine tool setting, and the ratio of roll are conducted by polynomial functions of fifth-order:

$$\begin{aligned}
\kappa &= c_{10} + c_{11} \cdot (\psi_{c1} - \psi_{c10}) + c_{12} \cdot (\psi_{c1} - \psi_{c10})^2 .... + c_{15} \cdot (\psi_{c1} - \psi_{c10})^5 \\
\mu &= c_{20} + c_{21} \cdot (\psi_{c1} - \psi_{c10}) + c_{22} \cdot (\psi_{c1} - \psi_{c10})^2 .... + c_{25} \cdot (\psi_{c1} - \psi_{c10})^5 \\
h_d &= c_{30} + c_{31} \cdot (\psi_{c1} - \psi_{c10}) + c_{32} \cdot (\psi_{c1} - \psi_{c10})^2 .... + c_{35} \cdot (\psi_{c1} - \psi_{c10})^5 \\
\Delta e &= c_{40} + c_{41} \cdot (\psi_{c1} - \psi_{c10}) + c_{42} \cdot (\psi_{c1} - \psi_{c10})^2 .... + c_{45} \cdot (\psi_{c1} - \psi_{c10})^5 \\
\Delta i_{g1} &= c_{50} + c_{51} \cdot (\psi_{c1} - \psi_{c10}) + c_{52} \cdot (\psi_{c1} - \psi_{c10})^2 .... + c_{55} \cdot (\psi_{c1} - \psi_{c10})^5
\end{aligned} \quad (2)$$

where ψ_{c1} is the angle of rotation of the imaginary generating crown gear in pinion tooth surface generation.

Therefore, the maximum tooth contact pressure, maximum transmission error, EHD load carrying capacity, and friction factor depend on 33 manufacture parameters:

$$
\begin{aligned}
p_{\max}(mp) &= p_{\max}\left(\begin{array}{l} r_{prof1}, r_{prof2}, \\ \Delta r_{t0}, \sum_{i=1}^{i=5}\sum_{j=0}^{j=5} c_{ij} \end{array}\right) \\
\Delta\varphi_{2\max}(mp) &= \Delta\varphi_{2\max}\left(\begin{array}{l} r_{prof1}, r_{prof2}, \\ \Delta r_{t0}, \sum_{i=1}^{i=5}\sum_{j=0}^{j=5} c_{ij} \end{array}\right) \\
W(mp) &= W\left(\begin{array}{l} r_{prof1}, r_{prof2}, \\ \Delta r_{t0}, \sum_{i=1}^{i=5}\sum_{j=0}^{j=5} c_{ij} \end{array}\right) \\
f_T(mp) &= f_T\left(\begin{array}{l} r_{prof1}, r_{prof2}, \\ \Delta r_{t0}, \sum_{i=1}^{i=5}\sum_{j=0}^{j=5} c_{ij} \end{array}\right)
\end{aligned}
\tag{3}
$$

2.2. The Optimization of Machine Tool Settings and Head-Cutter Geometry

An optimization method is applied to systematically define optimal head-cutter geometry and machine tool settings to simultaneously minimize maximum tooth contact pressure and angular displacement error of the driven gear, to maximize the EHD load carrying capacity of the oil film, and to minimize power losses in the oil film. The proposed optimization procedure relies heavily on the loaded tooth contact analysis for the prediction of maximum tooth contact pressure and transmission errors and on the EHD lubrication analysis to calculate the EHD load carrying capacity of the oil film, and the friction factor. The employed methods are developed in Refs. [5 - 8]

The goal of the optimization is to minimize tooth contact pressure and transmission errors, to maximize the EHD load carrying capacity of the oil film, and to minimize power losses in the oil film while keeping the loaded contact pattern inside the physical tooth boundaries of the pinion and the gear. The applicable objective functions can be expressed as

$$
f(mp) = c_p \cdot \frac{p_{\max}(mp)}{p_{\max 0}} + c_\varphi \cdot \frac{\Delta\varphi_{2\max}(mp)}{\Delta\varphi_{2\max 0}} \tag{4}
$$

$$
f(mp) = c_w \cdot \frac{W(mp)}{W_0} + c_f \cdot \frac{f_T(mp)}{f_{T0}} \tag{5}
$$

where $p_{\max 0}$ and $\Delta\phi_{2\max 0}$ are the maximum tooth contact pressure and transmission error, W_0 and f_{T0} are EHD load carrying capacity of the oil film and the friction factor obtained for the initial values of manufacture parameters; c_p, c_ϕ, c_W, and c_f are non-negative weight coefficients, expressing their relative importance.

The proper constraints are due to the requirements that the contact pattern remains inside the possible contact area defined by load distribution calculation and inside the physical tooth boundaries of the pinion and the gear. It leads to the requirement that the contact load outside the instantly possible contact area should be zero. Therefore, the constraint can simply be denoted by

$$
C(mp) = 0 \tag{6}
$$

where C is the total of tooth surface points with instantaneously not existing contact loads. Therefore, it depends on the tooth surface topography through the manufacture parameters ***mp***.

The optimization problem formulated according to Eqs. (4), (5), and (6) is a nonlinear constrained optimization problem. Functions $f(\boldsymbol{mp})$ and $C(\boldsymbol{mp})$ are not available analytically, they exist numerically through the load distribution calculation and EHD lubrication analysis. Therefore, the computer simulation of load distribution and EHD lubrication must be run, repeatedly, in order to compute the various quantities needed by the optimization algorithm. The load distribution calculation and the EHD lubrication analysis are based on highly nonlinear systems of equations. An approximate and iterative technique is used to perform the load distribution calculation and the EHD lubrication analysis. This causes that the calculation of partial derivatives for gradient-based optimization algorithms to be quite impractical. For this reason, a nonderivative method is selected to solve this particular optimization problem. Here, the pattern search method is used.

3. Manufacture of Face-Hobbed Hypoid Gears on CNC Hypoid Generator

The CNC machine for generation of spiral bevel and hypoid gears is provided with six degrees-of-freedom for three rotational motions (θ, ζ, η), and three translational motions (*X*, *Y*, *Z*, Fig. 5). The six axes of CNC generator are directly driven by the servo motors and able to implement prescribed functions of motions. The face-hobbing method requires simultaneous six-axis control (the face-milling method requires only five-axis control). The following coordinate systems are applied to describe the relations and motions in the CNC generator (Fig. 2): Coordinate systems $K_t(x_t, y_t, z_t)$ and $K_i(x_i, y_i, z_i)$ are rigidly connected to the head-cutter and the pinion/gear, respectively. The coordinate transformation from system K_t to system K_i performs the following equation:

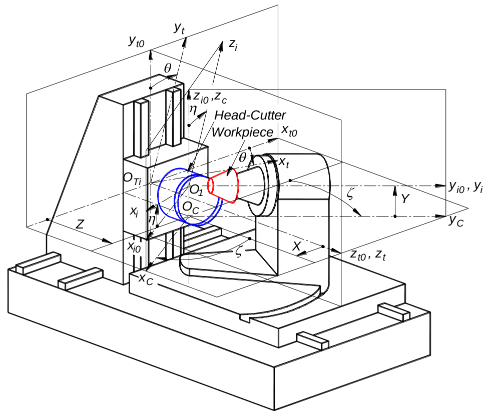

Fig. 2 Machine-tool setting for pinion tooth-surface finishing on CNC gnerator

$$\vec{\mathbf{r}}_i = \mathbf{M}_i(\eta)\cdot\mathbf{M}_{ti0}(\zeta, X, Y, Z)\cdot\mathbf{M}_t(\theta)\cdot\vec{\mathbf{r}}_t = \mathbf{M}_{ti}^{CNC}\cdot\vec{\mathbf{r}}_t \tag{7}$$

The location and the orientation of the tool with respect to the pinion/gear are given in coordinate systems that are represented for a conventional, cradle-type generator (Fig. 1). An algorithm is developed for the execution of motions on the CNC generator using the relations valid for the cradle-type machine. This algorithm is based on the conditions that the relative position of the axes of the head-cutter and the pinion rotations, z_{t0} and y_{i0}, and the axial relative position of the head-cutter and the pinion/gear should be the same whether the pinion/gear is cut on a cradle-type or on a CNC hypoid generator.

To ensure the same relative position of the two axes, z_{t0} and y_{i0}, on both the cradle-type and CNC hypoid generating machines, the elements of the coordinate transformation matrices should be equal. On the basis of Eqs. (1) and (7) the following condition should be satisfied:

$$\begin{aligned}\mathbf{e}_{i0}^{(z_{t0})} &= \mathbf{M}_{i2}\cdot\mathbf{M}_{i1}\cdot\mathbf{M}_{c4}\cdot\mathbf{M}_{c3}\cdot\mathbf{M}_{c2}\cdot\mathbf{e}_{t0}^{(z_{t0})}\\ &= \mathbf{M}_{ti0}(\zeta, X, Y, Z)\cdot\mathbf{e}_{t0}^{(z_{t0})}\end{aligned} \tag{8}$$

The same relative position of the head-cutter and the pinion along their axes in the case of both machines, is satisfied by applying the following condition

$$\begin{aligned}\vec{\mathbf{r}}_{i0}^{(O_t)} &= \mathbf{M}_{i2}\cdot\mathbf{M}_{i1}\cdot\mathbf{M}_{c4}\cdot\mathbf{M}_{c3}\cdot\mathbf{M}_{c2}\cdot\vec{\mathbf{r}}_{t0}^{(O_t)}\\ &= \mathbf{M}_{ti0}\cdot\vec{\mathbf{r}}_{t0}^{(O_t)}\end{aligned} \tag{9}$$

4. Results and Discussion

A computer program was developed to implement the formulation provided above. By applying this program the optimal machine tool settings were calculated and functions were developed for the execution of motions on the CNC hypoid generator using the relations on the cradle-type machine.

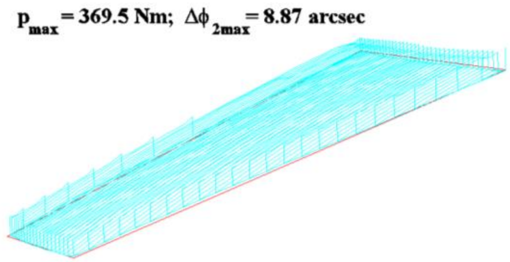

Fig. 3 Tooth contact pressure distribution in hypoid gear pair when the pinion and gear tooth surfaces are fully conjugate

The load distribution calculation was performed for 21 instantaneous positions of the pinion and the gear rolling through a mesh cycle. The tooth contact pressure distributions along the potential contact lines for 21 instantaneous positions and for all the adjacent tooth pairs engaged for a particular position of the mating members, for the case when no modifications are introduced into the pinion teeth of the hypoid gear pair, namely straight-lined head-cutter profile and the basic values of machine tool settings are applied, are shown in Fig. 3. In this case the pinion and gear tooth surfaces are fully conjugate. The obtained maximum tooth contact pressure is *369.5 MPa* and the maximum angular displacement error of the driven gear is *8.87 arcsec*. The tooth contact pressure distribution for the case when the pinion teeth are manufactured by the head-cutter of optimized geometry and by optimal variation in machine tool settings governed by Eq. (2) is shown in Fig. 4. It can be observed that the maximum tooth contact pressure is reduced to $p_{max} = 332.4\,MPa$ and the maximum transmission error to $\Delta\phi_{2\,max} = 0.79\,arc\,sec$. Similar reductions in the maximum tooth contact pressure and in the maximum displacement error of the driven gear are obtained in the case of a spiral bevel gear pair (Figs. 5 and 6).

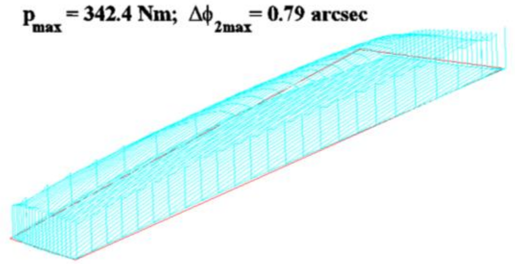

Fig. 4 Tooth contact pressure distribution in hypoid gear pair when the pinion tooth is manufactured by the head-cutter of optimized geometry and by optimal variation in machine tool settings

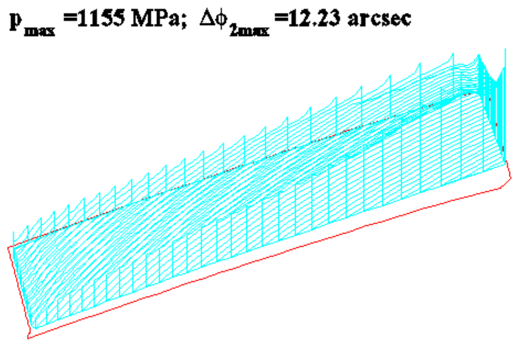

Fig. 5 Tooth contact pressure distribution in the spiral bevel gear pair when the pinion and gear tooth surfaces are fully conjugate

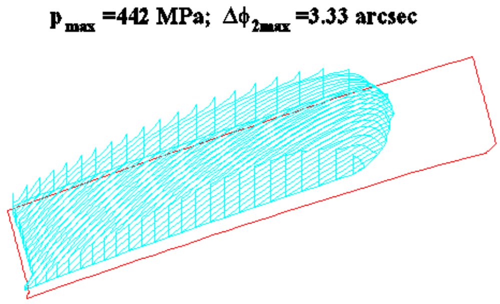

Fig. 6 Tooth contact pressure distributions along the potential contact lines when the pinion tooth is manufactured by optimized head-cutter and machine tool settings

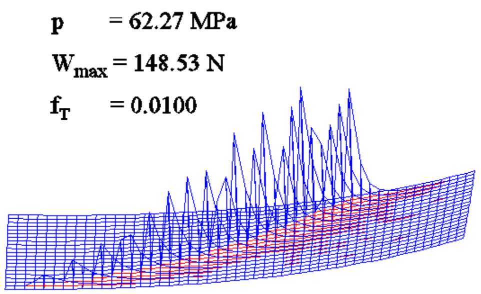

Fig. 7 Pressure distribution in the oil film in spiral bevel gear pair for the basic values of machine tool setting parameters

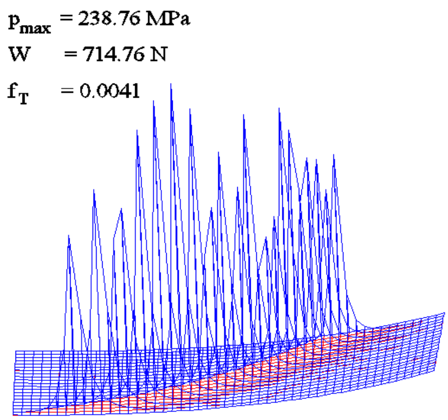

Fig. 8 Pressure distribution in the oil film in the spiral bevel gear pair for the optimal values of machine tool setting parameters

By applying the optimal combination of head-cutter geometry and machine tool settings the lubrication performances of the spiral bevel gear pair are improved. In Figs. 7 and 8 it can be considered that there is a considerable increase in the EHD load carrying capacity and reduction in the power losses in the oil film.

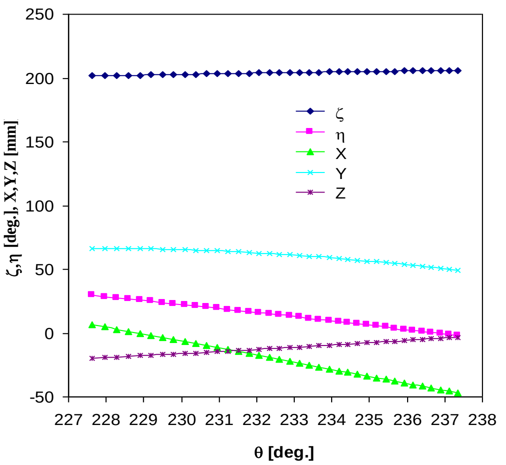

Fig. 9 Motion graphs for the CNC hypoid generator for finishing the pinion in function of the rotation angle of the head-cutter on the CNC generator

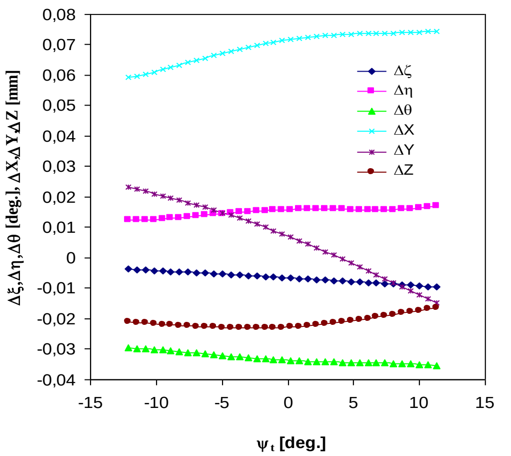

Fig. 10 Differences in motions on the CNC hypoid generator as results of using head-cutter of optimized geometry, optimal polynomial functions for the conduction of variation in machine tool settings and modified roll for pinion tooth flank generation

The graph shown in Fig. 9, represent the execution of motions on the CNC hypoid generator for finishing the pinion teeth governed by Eq. (2). The variation in motion parameters is expressed in function of the rotation angle of the head-cutter on the CNC generator. The differences in the values of motion parameters on the CNC hypoid generator, as results of using optimal polynomial functions for the conduction of variation in machine tool settings and modified roll for pinion tooth flank generation, are shown in Fig. 10.

5. Conclusions

An advanced method for the manufacture of spiral bevel and hypoid gears on CNC hypoid generator is presented. The optimal head-cutter geometry and machine tool settings are determined to introduce the optimal tooth modifications into the teeth of spiral bevel and hypoid gears in order to reduce the tooth contact pressure and transmission errors, to maximize the EHD load carrying capacity of the oil film, and to minimize power losses in the oil film. The method is based on machine tool setting variation on the cradle-type generator conducted by polynomial functions of fifth-order. An algorithm is developed for the execution of motions on the CNC hypoid generator using the optimal relations on the cradle-type machine. By applying the head-cutter of optimal geometry and the optimal variation in machine tool settings the following operating parameters are improved:

(1) In the case of the hypoid gear pair moderate reduction in the maximum tooth contact pressure of 10% and a drastic reduction in the transmission errors of 91% were obtained.

(2) For the spiral bevel gear pair significant reductions in the maximum tooth contact pressure of 62% and in the transmission errors of 73% were achieved.

(3) The EHD load carrying capacity of the oil film is drastically increased for 252% and the power losses in the oil film are reduced for 61% in the case of the spiral bevel gear pair.

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