Advances in Technology Innovation, vol. 8, no. 2, 2023, pp. 150-161

State Estimation and Optimal Control of Four-Tank System with Stochastic Approximation Approach

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Received 28 November 2022; received in revised form 20 February 2023; accepted 26 February 2023

DOI: https://doi.org/10.46604/aiti.2023.11232

Abstract

This study aims to optimally control the level of a four-tank system at the steady state in the random disturbance environment using the stochastic approximation (SA) approach. Firstly, the stochastic optimal control problem of an equivalent discrete-time is introduced, where the voltages to the pumps are the control inputs. By minimizing the sum of squared errors, the liquid levels are estimated. Then, first-order necessary conditions are derived by defining the Hamiltonian function. Thus, the optimal voltages are calculated based on the estimated liquid levels to update the gradient of the cost function. Finally, for illustration, parameters in the system are considered and a simulation is conducted. The simulation results show that the state estimation and control law design can perform well, and the liquid levels are addressed along the steady state. In conclusion, the applicability of the SA approach for handling a four-tank system with random disturbances is demonstrated.

Keywords: four-tank system, nonlinear optimal control, stochastic approximation, state estimation

1. Introduction

Water tank systems have been widely studied in engineering, such as mechanical and chemical systems. In particular, a four-tank system, which is a standard nonlinear dynamical system, has a more complex liquid-level control problem than two- and three-tank systems. Considering the interconnected structure between the inputs and outputs in these four tanks, changing a single input to control the system output is complex, and the stabilization of the system changes once a single input is changed. The four-tank system with coupling effects between the input and output causes complex nonlinear behavior, and this nonlinear process is troublesome to manage. Thus, it is challenging to stabilize the response output from a four-tank system that is a multiple-input multiple-output (MIMO) process [1-2]. Moreover, most industrial processes have the problem of controlling the liquid level in the tanks. Typically, liquids used in chemical plants and in mixing treatments in tanks must be controlled at a steady-state height, and the flow between tanks should be regulated [3] at the desired level. The various applications of liquid-level control, including nuclear power plants [4], food processing [5], and beverages and pharmaceuticals [6], have been well-presented in the literature.

For simplicity, a four-tank system consisting of four interconnected tanks and two pumps are considered. This system aims to control the liquid level in the two lower tanks using two pumps. The inputs of the process are the voltage of the pumps, and the outputs are the liquid levels in the lower two tanks [7-8]. These two pumps convey liquid from a basin to four overhead

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tanks. The liquid is freely drained from the two upper-level tanks into the two bottom-level tanks. Then, the liquid levels in the two bottom tanks are measured. In such a piping system, each pump affects the liquid levels in both the measured tanks. A portion of the flow from the pumps is directed to one of the tanks at the bottom level, while the rest of the flow from the pumps is directed to the overhead tank, which drains freely into the other tanks at the bottom level. The amount of flow between the inputs and the outputs can vary by adjusting the bypass valves of the system [9-10].

Several practical studies have been conducted on the control of the four-tank systems. Proportional-integral-derivative (PID), model predictive control (MPC), and fuzzy modified model reference adaptive control (FMMRAC) were implemented to control a four-tank system [11]. The system's performance was analyzed using conventional proportional-integral (PI) and hybrid fuzzy PI controllers. Linear quadratic regulator (LQR), linear quadratic Gaussian regulator (LQGR), and H_2 and H_∞ controllers [12] were applied to manage the four-tank system separately. Then, compare their performances in terms of disturbance rejection to study the effects of these control systems on the four-tank system. Moreover, a distributed control and estimation method [13] was designed for a multivariate four-tank process, whereas active disturbance rejection techniques [14] and disturbance observers [15] were proposed for studying four-tank systems in the presence of random disturbances.

Furthermore, some studies have been conducted on tank systems using the simultaneous perturbation stochastic approximation (SPSA) method. For example, the water cooling of sulfuric acid in a two-tank system was studied using the SPSA technique, where a neural model and a model-based predictive neural PID controller were developed [16]. A novel optimization method based on the SPSA approach was proposed to maximize the control performance of steam-generator level control [17]. The SPSA method was also developed and implemented on a dual-tank liquid-level control system to improve the performance of a PID control system for parameter optimization in the tank model and actual equipment [18]. In addition, the SPSA method was applied to optimize the trajectories of good placement in the reservoir management optimization problem [19]. However, few studies used the SA technique to manage four-tank systems.

Therefore, this study aims to explore the application of the SA approach [20] for controlling a four-tank system with random disturbances, which might occur from measurement and human errors. Since the actual state of the system, which is the water level in the tank, is uncertain in the presence of random disturbances, estimating the water level in the tank will be difficult. In this situation, researchers are motivated to propose a computational approach based on the SA approach to handle the state estimation of the system and control the water level at a steady state as the main contribution to this study. This computational algorithm is known as the stochastic approximation for state-control (SASC) algorithm.

Firstly, a loss function is defined, where the differences between the actual and estimated output are minimized. Then, the SA updating rule is derived for obtaining state estimates after the first-order necessary condition is satisfied. In addition, a stochastic optimization problem is introduced to minimize the cost function when a sequence of control inputs can be determined to stabilize the system. Thus, the Hamiltonian function is defined, and the optimality conditions are derived. On this basis, the control law is designed through the SA updating rule based on state estimates. In this study, the performance of the SASC algorithm is given by mean squared errors (MSE) for state estimation and the cost function for the control effort. For illustration, the values of parameters in the system are considered, and the four-tank system's discrete-time stochastic optimal control problem is solved iteratively using the SASC algorithm. Finally, the simulation results are presented and discussed after being compared with the results from the extended Kalman filter (EKF).

The rest of this paper is organized as follows. Section 2 describes the mathematical model of a four-tank system and defines the system's discrete-time nonlinear stochastic optimal control problem. Section 3 explains state estimation and control law design using the SA algorithm, and the iterative procedure with the SA updating scheme is summarized as the SASC algorithm. Section 4 presents and discusses simulation results obtained using the SASC algorithm. Finally, Section 5 presents a concluding remark.

2. Problem Description

Consider a four-tank system [2-3] consisting of four connected tanks and two pumps, as shown in Fig. 1. Pump 1 extracts liquid from the basin below and pumps it to tanks 1 and 4, whereas pump 2 pumps liquid from the basin below to tanks 2 and 3. Denote l_1, l_2, l_3, and l_4 as the liquid level of tanks 1, 2, 3, and 4, respectively, while γ_1 and γ_2 are the flow parameters of pumps 1 and 2.

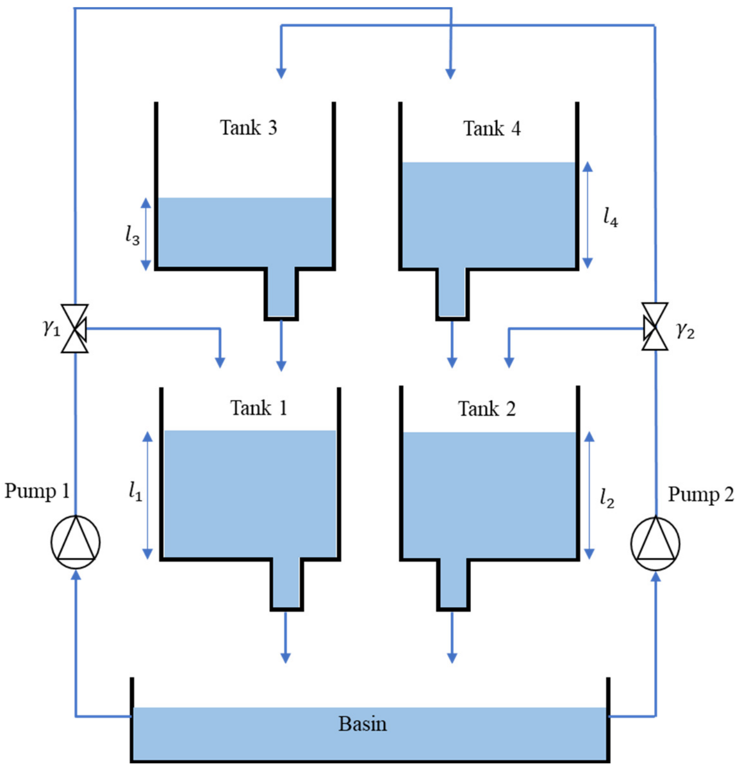


Fig. 1 Four tanks system

Suppose q_{in1}, q_{in2}, q_{in3}, and q_{in4} are the flows into tanks 1, 2, 3, and 4, respectively, while q_{pump1} and q_{pump2} represent the flow out of the electrical pumps 1 and 2. The flows into tanks 1 and 4 are equal to the flow out of electrical pump 1, whereas those into tanks 2 and 3 are equal to electrical pump 2. The relations between the flows at each outlet pipe and the total flows from pumps 1 and 2 depend on the flow parameters γ_1 and γ_2 are given as follows [6],

$$q_{pump1} = k_1 v_1 \tag{1}$$

$$q_{pump2} = k_2 v_2 \tag{2}$$

$$q_{in1} = k_1 v_1 \gamma_1 \tag{3}$$

$$q_{in2} = k_2 v_2 \gamma_2 \tag{4}$$

$$q_{in3} = k_2 v_2 (1-\gamma_2) \tag{5}$$

$$q_{in4} = k_1 v_1 (1-\gamma_1) \tag{6}$$

where k_1 and k_2 are the pump coefficients, while v_1 and v_2 are the input voltages to the pumps. The flow out of tanks is given by Torricelli's principle by using Bernoulli's equation [7],

$$q_{out1} = a_1 \sqrt{2gl_1} \tag{7}$$

$$q_{out2} = a_2\sqrt{2gl_2} \tag{8}$$

$$q_{out3} = a_3\sqrt{2gl_3} \tag{9}$$

$$q_{out4} = a_4\sqrt{2gl_4} \tag{10}$$

where a_1, a_2, a_3, and a_4 are the cross-section of the outlet pipe of tanks 1, 2, 3, and 4, while g is the acceleration due to gravity.

The mathematical model of the four-tank system is described by the mass balance equation [11-12] given by

Tank 1:

$$A_1\frac{dl_1}{dt} = q_{in1} + q_{out3} - q_{out1} \tag{11}$$

Tank 2:

$$A_2\frac{dl_2}{dt} = q_{in2} + q_{out4} - q_{out2} \tag{12}$$

Tank 3:

$$A_3\frac{dl_3}{dt} = q_{in3} - q_{out3} \tag{13}$$

Tank 4:

$$A_4\frac{dl_4}{dt} = q_{in4} - q_{out4} \tag{14}$$

where A_1, A_2, A_3, and A_4 are the area of the cross-section of tanks 1, 2, 3, and 4, respectively. Eqs. (11)-(14) show that the net change in the volume in a tank is equal to the difference between the volume entering and leaving the tank. By substituting Eqs. (3)-(10) into Eqs. (11)-(14), the dynamics of the four-tank system are given as follows,

$$\frac{dl_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gl_1} + \frac{a_3}{A_1}\sqrt{2gl_3} + \frac{\gamma_1 k_1}{A_1}v_1 \tag{15}$$

$$\frac{dl_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gl_2} + \frac{a_4}{A_2}\sqrt{2gl_4} + \frac{\gamma_2 k_2}{A_2}v_2 \tag{16}$$

$$\frac{dl_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gl_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \tag{17}$$

$$\frac{dl_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gl_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \tag{18}$$

Define the state variable $x = (x_1, x_2, x_3, x_4)^T$ with $x_1 = l_1$, $x_2 = l_2$, $x_3 = l_3$, and $x_4 = l_4$, and let the control variable $u = (u_1, u_2)^T$ with $u_1 = v_1$ and $u_2 = v_2$, the dynamics of the four-tank system in Eqs. (15)-(18) can be formulated in its equivalent discrete-time state equation [21],

$$x(k+1) = f\left[x(k), u(k)\right] \tag{19}$$

where $f = (f_1, f_2, f_3, f_4)^T: \mathfrak{R}^4 \times \mathfrak{R}^2 \to \mathfrak{R}^4$ is the system dynamics given by

$$f_1 = x_1 + \tau\left(-\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} + \frac{\gamma_1 k_1}{A_1}u_1\right) \tag{20}$$

$$f_2 = x_2 + \tau\left(-\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} + \frac{\gamma_2 k_2}{A_2}u_2\right) \tag{21}$$

$$f_3 = x_3 + \tau\left(-\frac{a_3}{A_3}\sqrt{2gx_3} + \frac{(1\text{-}\gamma_2)k_2}{A_3}u_2\right) \tag{22}$$

$$f_4 = x_4 + \tau\left(-\frac{a_4}{A_4}\sqrt{2gx_4} + \frac{(1\text{-}\gamma_1)k_1}{A_4}u_1\right) \tag{23}$$

with τ as the sampling time. The output variable $y = (y_1, y_2)^T$ is defined by

$$y(k) = h\left[x(k)\right] \tag{24}$$

where the output measurement channel $h = (h_1, h_2)^T: \Re^4 \to \Re^2$ is

$$h_1 = x_1 \tag{25}$$

$$h_2 = x_2 \tag{26}$$

which represents the solution for the liquid level of tank 1 and 2 as $l_1(t)$ and $l_2(t)$, respectively. Therefore, the discrete-time system [22] consists of Eqs. (19) and (24), which are disturbed by random noises, are denoted as

$$x(k+1) = f\left[x(k), u(k)\right] + G\omega(k) \tag{27}$$

$$y(k) = h\left[x(k)\right] + \eta(k) \tag{28}$$

where G is a 4×4 coefficient matrix, whereas $\omega(k) \in \mathcal{R}^4$, $k = 0,1,\cdots,N-1$, and $\eta(k) \in \mathcal{R}^2$, $k = 0,1,\cdots,N$, are the additive Gaussian white noises with zero mean, and their respective covariance matrices are given by $Q_\omega \in \mathcal{R}^{4\times4}$ and $R_\eta \in \mathcal{R}^{2\times2}$.

$$x(0) = x_0 \tag{29}$$

The initial state Eq. (29) is a random vector with the mean and state error covariance matrix are, respectively, given by

$$E(x_0) = \bar{x}_0 \tag{30}$$

$$E\left[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\right] = M_0 \tag{31}$$

where $M_0 \in \mathcal{R}^{4\times4}$ is a positive definite matrix and $E[\quad]$ is the expectation operator. It is assumed that the initial state, process noise, and measurement noise are statistically independent. Here, the aim is to determine a set of admissible control sequences $u(k) \in \mathcal{R}^2$, $k = 0,1,\cdots,N-1$, such that the following expected cost function

$$J(u) = E\left\{\varphi\left[x(N)\right] + \sum_{k=0}^{N-1} L\left[x(k), u(k)\right]\right\} \tag{32}$$

is minimized over the dynamical system given in Eqs. (27) and (28), where $\varphi: \Re^4 \to \mathcal{R}$ is the terminal cost and $L: \Re^4 \times \Re^2 \to \mathcal{R}$ is the cost under summation. This problem is known as the discrete-time nonlinear stochastic optimal control problem for the four-tank system and is referred to as problem (P).

3. Stochastic Approximation (SA) Approach

Now, consider the following recursive equation,

$$\theta^{i+1} = \theta^i - a_i \times g^i \tag{33}$$

where θ^i is the set of parameters to be estimated, $g^i = g(\theta^i)$ is the stochastic gradient, and a_i is the gain sequence. This equation is known as the SA approach [20]. Thus, the state estimation and the optimal control design based on the SA approach will be further discussed.

3.1. State estimation

Consider the state mean propagation [23-24] for Eqs. (27) and (28), given by

$$\bar{x}(k+1) = f\left[\bar{x}(k), u(k)\right] \tag{34}$$

$$\bar{y}(k) = h\left[\bar{x}(k)\right] \tag{35}$$

where $\bar{x}(k)$ and $\bar{y}(k)$ are the expected state sequence and the expected output sequence, respectively. To find the optimal state estimate, introduce the following weighted least-squares problem [21-22],

$$\begin{aligned}\min_x J_{sse}(x) = &\frac{1}{2}\left\{\left[x(k)-\bar{x}(k)\right]^T (M_0)^{-1}\left[x(k)-\bar{x}(k)\right]\right\}\\ &+\frac{1}{2}\left\langle\left\{y(k)-h\left[x(k)\right]\right\}^T (R_\eta)^{-1}\left\{y(k)-h\left[x(k)\right]\right\}\right\rangle\end{aligned} \tag{36}$$

where J_{sse} is the sum of squares error, M_0 is the initial state error covariance, and R_η is the output noise covariance. By taking the first-order derivative, the gradient of the sum squares of errors is defined by

$$\nabla_x J_{sse}(x) = (M_0)^{-1}\left[x(k)-\bar{x}(k)\right] - C^T (R_\eta)^{-1}\left\{y(k)-h\left[x(k)\right]\right\} \tag{37}$$

where $C = \partial h/\partial x$. Thus, by using the SA approach [20] in Eq. (33), the optimal state estimate is obtained from

$$\hat{x}(k)^{i+1} = x(k)^i - a_{1,i} \times \nabla_x J_{sse}\left[\hat{x}(k)^i\right] \tag{38}$$

where $a_{l,i} > 0$ is the learning rate and the optimal output estimate is determined by

$$\hat{y}(k)^i = h\left[\hat{x}(k)^i\right] \tag{39}$$

Remarks: The state estimate $\hat{x}$ is the optimal state estimate, which minimizes the sum of squares of error J_{sse}. The norm of the state estimate $\hat{x}$ and the state mean $\bar{x}$ is relatively close within a small tolerance. Using the SA approach for state estimation does not need the state error covariance matrix equation as derived in the Kalman filtering approach [21-22].

3.2. Optimality conditions

Refer to the problem (P), the expected cost function [24] in Eq. (32) can be defined by

$$J(u) = \varphi\left[\bar{x}(N)\right] + \sum_{k=0}^{N-1} L\left[\bar{x}(k), u(k)\right] \tag{40}$$

and the state propagation from Eq. (34) is considered. Define the Hamiltonian function [19],

$$H(u) = L\left[\bar{x}(k), u(k)\right] + p(k+1)^T f\left[\hat{x}(k), u(k)\right] \tag{41}$$

where $p(k) \in \mathcal{R}^4$, $k = 0, 1, \cdots, N$ is the costate sequence to be determined later. Thus, the augmented cost function becomes

$$J'(u) = \varphi\left[\bar{x}(N)\right] + \sum_{k=0}^{N-1} H(k) - p(k+1)^T \bar{x}(k+1) \tag{42}$$

Examining the increment in the augmented cost function J' due to increments in all variables, which are $\bar{x}(k)$, $p(k)$, and $u(k)$ According to the Lagrange multiplier theory, this increment dJ' should be zero at a constrained minimum [21-22]. Thus, taking the first-order derivative of the augmented cost function and Hamiltonian function, the following optimality conditions are derived.

(a) Stationary condition

$$\nabla_{u(k)} L\left[\bar{x}(k), u(k)\right] + \nabla_{u(k)} f\left[\hat{x}(k), u(k)\right]^T p(k+1) = 0 \tag{43}$$

(b) State equation

$$\bar{x}(k+1) = f\left[\hat{x}(k), u(k)\right] \tag{44}$$

(c) Costate equation

$$p(k) = \nabla_{x(k)} L\left[\bar{x}(k), u(k)\right] + \nabla_{x(k)} f\left[\hat{x}(k), u(k)\right]^T p(k+1) \tag{45}$$

(d) Output equation

$$\bar{y}(k) = h\left[\bar{x}(k)\right] \tag{46}$$

(e) Boundary conditions

$$\hat{x}(0) = \bar{x}_0 \tag{47}$$

$$p(N) = \nabla_{x(N)} \varphi\left[\bar{x}(N)\right] \tag{48}$$

Remark: For the sake of convenience, the following standard cost function in quadratic criterion [23-24] could be calculated

$$\phi\left[\bar{x}(N)\right] = \frac{1}{2}\bar{x}(N)^T S(N)\bar{x}(N) \tag{49}$$

$$L\left[\bar{x}(k), u(k)\right] = \frac{1}{2}\left[\bar{x}(k)^T Q\bar{x}(k) + u(k)^T Ru(k)\right] \tag{50}$$

when a proper cost function is not provided. Thus, the necessary conditions are simple.

3.3. Optimal control design

Define an equivalent stochastic optimization problem [20] to problem (P), and this problem is regarded as the problem (Q), given by

$$Minimize \rightarrow J'(u) \tag{51}$$

where the necessary conditions in Eqs. (44) and (45) are satisfied. Hence, solving the problem (Q) would allow the design of the control law. By this, the gradient of the objective function in Eq. (42) is expressed by

$$\nabla_u J'(u) = \nabla_u H(k) \tag{52}$$

where

$$\nabla_u H = \nabla_u L\left[\bar{x}(k), u(k)\right] + \nabla_u f\left[\hat{x}(k), u(k)\right]^T p(k+1) \tag{53}$$

and the necessary condition for the problem (Q) is given by Eq. (43). Hence, the control law is updated from

$$u(k)^{i+1} = u(k)^i - a_{2,i} \times \nabla_u J'\left[u(k)^i\right] \tag{54}$$

where $a_{2,i} > 0$ is the learning rate. Here, the principle of separation is assumed to be satisfied when applying the SA approach [20] for state estimation and optimal control design. This statement is true for solving stochastic optimal control problems.

3.4. SA for state-control algorithm

From the discussion above, the computational procedure for applying the SA approach to state estimation and control law design is summarized as an iterative algorithm named the SASC algorithm. The steps of the SASC algorithm are as follows:

Data: Given f, h, G, φ, L, N, M_0, Q_ω, R_η, a_1, a_2, and y.

Step 0: Determine the initial control $u(k)^0 = u_0$ for $k = 0,1,\cdots,N-1$ and the initial state $\hat{x}(k)^0 = x_0$ for $k = 0,1,\cdots,N$ Set the tolerance ε and the iteration $i = 0$.

Step 1: Calculate the sum of squares error $J_{sse}[\hat{x}(k)^i]$ from Eq. (36), and the stochastic gradient $\nabla_x J_{sse}[\hat{x}(k)^i]$ from Eq. (37), respectively.

Step 2: Update the state estimate $\hat{x}(k)^{i+1}$ from Eq. (38).

Step 3: Compute the output estimate $\hat{y}(k)^i$ from Eq. (39).

Step 4: Solve the state equation forward in time from Eq. (44) with the given initial state $\bar{x}_0$ to obtain the state solution $\bar{x}(k)^i$.

Step 5: Solve the costate equation backward in time from Eq. (45) with the given final costate $p(N)$ to provide the costate solution $p(k)^i$.

Step 6: Compute the output measurement $\bar{y}(k)^i$ from Eq. (46).

Step 7: Calculate the cost function $J[u(k)]^i$ from Eq. (42) and calculate the stochastic gradient $\nabla_u J[u(k)]^i$ from Eq. (53).

Step 8: Update the control law $u(k)^{i+1}$ from Eq. (54).

Step 9: Test the convergence. If $\hat{x}(k)^{i+1} = \hat{x}(k)^i$ and $u(k)^{i+1} = u(k)^i$ within a given tolerance ε, stop, else set the iteration $i = i + 1$ and go to Step 1.

Remarks: From the steps of the SASC algorithm, the initial value of the state and control can be set to a zero vector in Step 0. The state estimation procedure is implemented from Step 1 to Step 3, and the two-point boundary-value problem is solved from Step 4 to Step 5. While the optimal control law is designed from Step 7 to Step 8, and the appropriate stopping criteria for the iteration can be set in Step 9.

4. Simulation Results

Table 1 Parameters in problem (P)

System parameters	Values
The cross-sectional area of the outlet hole for the tanks	$a_1 = 0.071\ cm^2$, $a_2 = 0.057\ cm^2$, $a_3 = 0.071\ cm^2$, $a_4 = 0.057\ cm^2$
The cross-sectional area of the tanks	$A_1 = 28\ cm^2$, $A_2 = 32\ cm^2$, $A_3 = 28\ cm^2$, $A_4 = 32\ cm^2$
Pump proportionality constants	$k_1 = 3.14\ cm^3/V_s$, $k_2 = 6.29\ cm^3/V_s$
Flow coefficient of the pumps	$\gamma_1 = 0.35$, $\gamma_2 = 1.35$
Gravitational acceleration	$g = 981\ cm/s^2$
Initial liquid level	$x_1(0) = 10.43$, $x_2(0) = 15.98$, $x_3(0) = 6.6$, $x_4(0) = 9.57$
Estimation and control parameters	Values
Sampling time	$\tau = 0.1\ s$
Final time step	$N = 80$
Weighting matrices	$S_N = O_{4\times4}$, $Q = diag(10, 10, 10^5, 10^5)$, $R = 100I_{2\times2}$
Covariance matrices	$M_0 = 0.2I_{4\times4}$, $Q_\omega = 0.01I_{4\times4}$, $R_\eta = 0.01I_{2\times2}$

Consider the system parameters for the four-tank system [3, 9], and the estimation and control parameters defined for the problem (P) listed in Table 1. Using these parameters, an illustrative example of the simulation of the four-tank system is studied to demonstrate the practicality of the SASC algorithm. The simulation of this study is conducted in the environment of the GNU Octave 7.2.0.

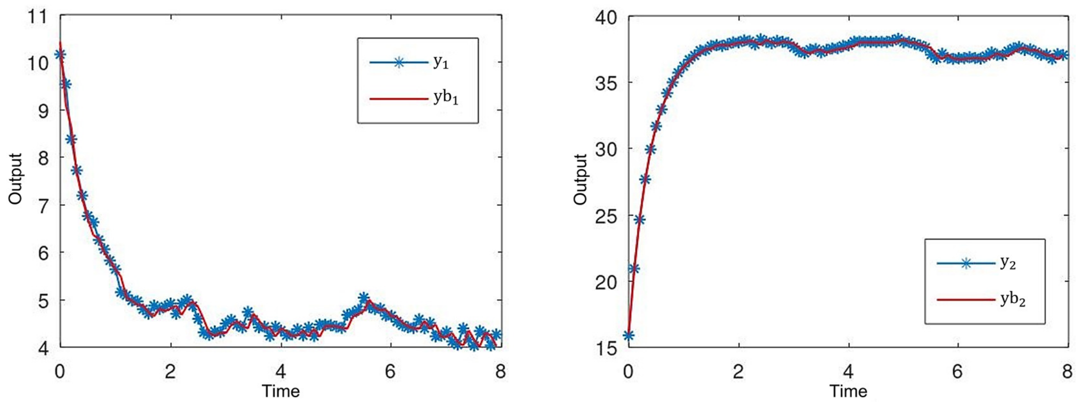
Table 2 presents the simulation results for controlling the four-tank system with random disturbances. The implementation of the SASC algorithm required 1,254 iterations to achieve convergence. For benchmark purposes, the results of the SASC algorithm were compared with results from the EKF algorithm since the EKF algorithm is the standard technique for nonlinear filtering and estimation.

The optimal cost of 3.4959×10^7 units obtained using the SASC algorithm was 1.15 % less than the optimal cost given by the EKF algorithm. This reduction demonstrated the practical application of the SASC algorithm in minimizing the cost function of the system. However, the performance of the SASC algorithm for state estimation, measured through the sum of squares error (SSE) and the MSE, was 98.7 % more accurate than that of the EKF algorithm. Therefore, the efficiency and accuracy of the SASC algorithm for solving the discrete-time nonlinear stochastic optimal control of the four-tank system were demonstrated.

Table 2 Simulation result

Algorithm	Optimal cost	SSE	MSE
EKF	3.5366×10^7	2.307386×10^{-1}	2.884233×10^{-3}
SASC	3.4959×10^7	3.091891×10^{-3}	3.864864×10^{-5}

Fig. 2 shows the final output trajectories when the SASC algorithm achieved convergence. The liquid level in tank 1, in which y_1 represents the actual liquid level and yb_1 is the estimated liquid level in tank 1, which was reduced from 10.43 cm and reached about 4.5 cm after 2 seconds at the final time of the iteration. While the liquid level in tank 2, in which y_2 represents the actual liquid level and yb_2 is the estimated liquid level in tank 2, which reached about 37.5 cm after 2 seconds at the final time of the iteration after increasing from 15.98 cm. In the random disturbance situation, it was challenging to maintain the steady state of the liquid level in tanks 1 and 2. However, the final liquid levels in both Tanks 1 and 2 were approximately determined in a satisfactory form by using the SASC algorithm.

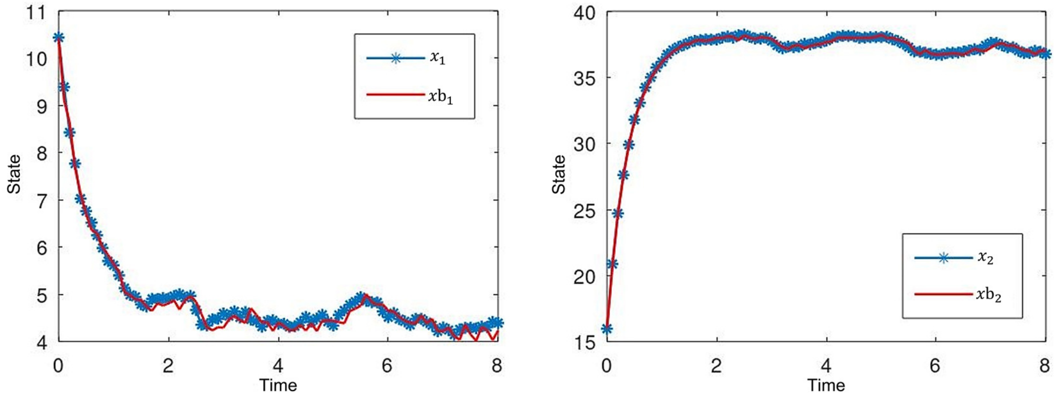


(a) Output trajectory y_1 – the liquid level in tank 1 (b) Output trajectory y_2 – the liquid level in tank 2

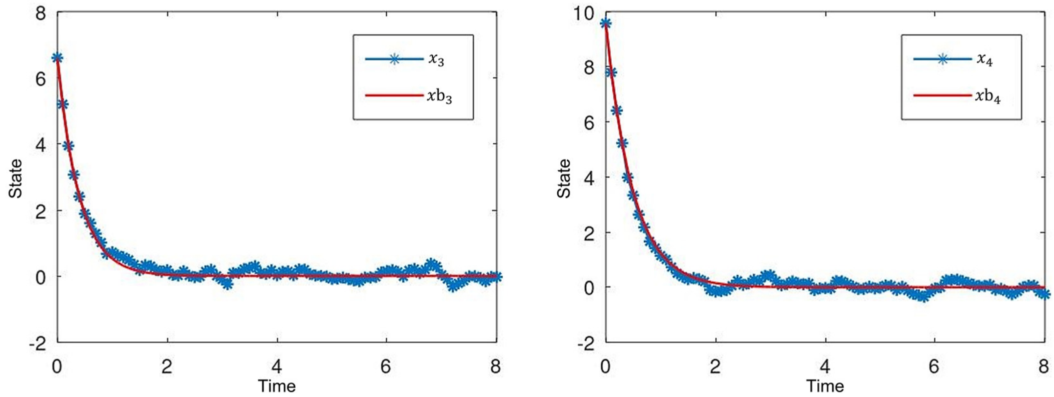
Fig. 2 Final output trajectories and real output trajectories

The final state trajectories are shown in Fig. 3. The liquid levels in tanks 1 and 2 exhibited fluctuation behaviors disturbed by random noise and were not easy to measure smoothly. Using the SASC algorithm, these trajectories were estimated acceptedly, and their trajectories were approximately measured. Conversely, the liquid levels in tanks 3 and 4 were unaffected by random disturbances. This is because their trajectories were smoothly predicted at their respective steady states approximately along with zero.

Fig. 4 shows the final control trajectories for regulating the liquid levels in the tanks. The optimal input voltage to pump 1 increased from -270 V to 0 V, and the optimal input voltage of pump 2 decreased from 182 V to 0 V. These control efforts effectively maintained liquid levels at approximately zero after 2 seconds. Therefore, the optimal solution to the four-tank problem was obtained satisfactorily when stationary conditions were satisfied, as shown in Fig. 5.



(a) State trajectory x_1 – the liquid level in tank 1 (b) State trajectory x_2 – the liquid level in tank 2



(c) State trajectory x_3 – the liquid level in tank 3 (d) State trajectory x_4 – the liquid level in tank 4

Fig. 3 Final state trajectory and real state trajectory

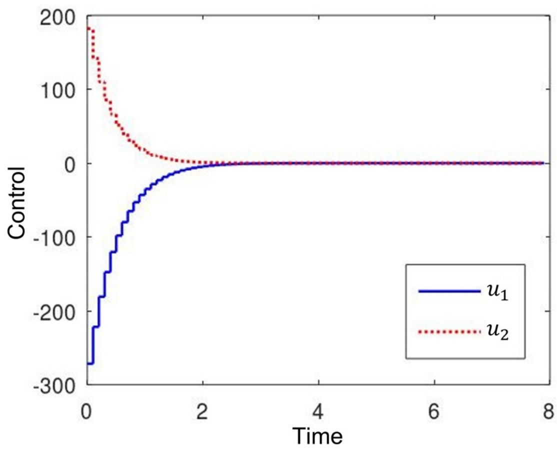


Fig. 4 Final control trajectories

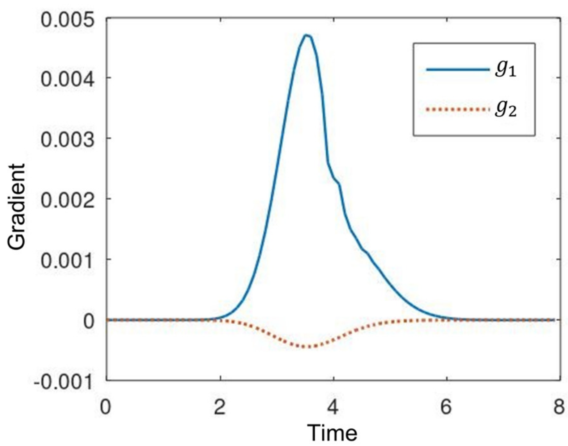


Fig. 5 Stationary conditions

5. Concluding Remarks

Optimizing and controlling the four-tank system with random disturbances through the SA approach were discussed in this study. Firstly, the discrete-time stochastic optimal control problem for the four-tank system was described by considering the presence of random disturbances. Subsequently, by applying the SA approach, the iterative algorithm, namely the SASC approach, was proposed to estimate the state dynamics and to design the optimal control law. Therefore, the state estimation of the system was satisfactorily handled, and the optimal control law, which was thoroughly designed based on the SA updating rule, was applied to minimize the performance index of the system. For illustration, researchers studied the control problem of a four-tank system with given parameters. The simulation results showed that the system was stabilized and controlled in a stochastic environment after using the SASC algorithm proposed. These results were also compared with results from the EKF technique, and a discussion was given. In conclusion, the efficiency and accuracy of the SASC algorithm are demonstrated. For future research, it is recommended to apply recent variants of the SA approach, like the Adam algorithm, for solving the stochastic optimal control problem of the four-tank system so that more acceptable results can be determined. In this way, the water level at the steady state will be identified in fewer iteration numbers, where the algorithm provides an iterative solution that can converge faster. Hence, the practicality and usefulness of the algorithm will be recommended.

Acknowledgment

This research was supported by the Universiti Tun Hussein Onn Malaysia (UTHM) through Tier 1 (vot Q121).

Conflicts of Interest

The authors declare no conflicts of interest.

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