Finite Element and Neural Network Based Predictive Model to Determine Natural Frequency of Laminated Composite Plates with Eccentric Cutouts under Free Vibration

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Abstract

This research proposes a predictive model to identify changes in the mechanical and geometrical properties of composite plates with eccentric cutouts based on natural frequency. Finite elements (FE) and neural networks are used to develop the model based on machine learning. First, the numerical analysis of free vibration is performed by the FE model on the laminated composite plates with a stacking sequence $[0/90]_{2s}$ under a clamped-free (CFFF) boundary condition. The outputs of the FE model (520 configurations) are then utilized to train the artificial neural network (ANN) model through the Levenberg-Marquardt method, and the developed ANN model is then used to evaluate the influence of various parameters on the natural frequency. The results show that the changes in the mechanical and geometrical properties of composite plates have impacts on the natural frequency. Furthermore, the findings of the ANN model are substantially identical to those of the numerical model, with a small margin of error.

1. Introduction

Laminated composite plates have been widely used in structural engineering due to their reduced weight, extended durability, and fatigue resistance. Because of these qualities, they are gaining attention in other engineering fields. Cutouts are generally used for ventilation, i.e., the passages for cables and fluids. They can have different shapes, e.g., square, circular, oval, or triangular shapes. However, their existence can have a major impact on the vibratory [1-3], static [4-6], and buckling [7-9] behavior of structures.

Concerning the vibration behavior of structures, Pham et al. [10] considered plates that are completely or partially in contact with fluid and analyzed them with isogeometric analysis (IGA) for free vibration. Pham et al. [11-12] used the finite element (FE) method to investigate the hygro-thermo-mechanical vibration of double-curved and functionally graded porous (FGP) sandwich plates as well as nanoplates made of functionally graded materials (FGM). Pham et al. [13] employed the ES-MITC3 element to analyze the free vibration of FGP annular-nanoplates with non-uniform thickness. In addition, Nguyen et al. [14] used the ES-MITC3 element to investigate the free vibration of FGP plates positioned on partially supported elastic foundations (PSEF). Rai [15] also used FE to examine the nonlinear behavior of reinforced concrete (RC) deep beams. Pham et al. [16] conducted a Monte Carlo simulation using FE analysis to evaluate the natural frequency of RC beams. The free vibration of FGP nanoplates lying on a two-parameter elastic media foundation was explored by Pham et al. [17].

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Recently, vibration response has emerged as an essential technique in structural health monitoring (SHM) [18]. A change in a structure's natural frequency is one of the indicators of a change in its mechanical or geometrical properties or of the presence of damage. Several researchers have investigated natural frequency to detect composite plate defects, such as delamination [19-20] and cracks [21-22].

Nowadays, the influence of cutouts on composite laminates is studied mainly using numerical analysis, with the FE analysis more specifically. Sivakumar et al. [23] presented a Ritz FE model to analyze the free vibration of laminates with cutouts. Ovesy and Fazilati [24] proposed two variants of the finite strip method (FSM) for analyzing the free vibration of composite plates with cutouts. Venkatachari et al. [25] used the extended FE technique to investigate the impact of environmental factors on the free vibration of structures. Boay [1] developed an FE method for calculating the free vibration of symmetric laminated composite plates with a central hole.

Artificial neural network (ANN) is an artificial intelligence technology widely used for prediction in a variety of engineering fields. Several studies in different fields are presented here. Truong et al. [26] integrated ANN with differential evolution (DE) to optimize the material distribution of bidirectional functionally graded (BFG) beams in free vibration. Yildirim [27] investigated the free vibration of axially functionally graded (AFG) and transversely functionally graded (TFG) beams using the ANN model. Furthermore, for FGM beams with varied gradation orientations and layer counts, the natural frequency was estimated using the FE approach. To analyze functionally graded annular plates under various boundary conditions, Jodaei et al. [28] applied both the differential quadrature and ANN techniques. Tran et al. [29] developed an ANN model to forecast the fundamental frequency of FGM plates by FE in a thermal environment using the ES-MITC3 element.

Due to the intricacy of laminated composites with cutouts, artificial intelligence was introduced to detect changes in natural frequency (i.e., mechanical and geometrical changes in the structures). Reddy [30] proposed a method for predicting the natural frequency of laminated composite plates using ANN under clamped boundary conditions. Altabey [31] predicted changes in the natural frequency of plates supported elastically. Timchenko and Osetrov [32] proposed convolutional neural networks (CNN) for predicting the natural frequency of composite plates. ANN was also used to evaluate the environmental effect on the vibrational response of a skew composite laminated sandwich plate [33].

Based on the findings of previous studies, this work aims to expand the use of the FE model to analyze the free vibration of nonlinear layered plates with eccentric square cutouts under clamped-free (CFFF) boundary conditions. The study of several parameters, such as cutout size ratio (d/a), number of cutouts, modular ratio (E1/E2), length-to-width ratio (a/b), and thickness ratio (h/a), is carried out using the FE software. The results of the FE model are used to develop an ANN model to predict natural frequency. The strategy is to use 240 data points to train, test, and validate the developed model. The study is limited to the first two modes of vibration. The influence of cutout size, number, and modular ratio on the natural frequency is then studied in a more general scope.

2. The FE Model

2.1. *Free vibration analysis*

The free vibration analysis of structures requires the solution of the following equation, called the eigenvalue problem. The frequency of plates can be obtained easily by the solution of the standard characteristic equation:

$$[K]\{\textstyle\prod\} - \omega^2[M]\{\textstyle\prod\} = \{0\} \tag{1}$$

where $\{\textstyle\prod\}$ denotes the transverse displacement vector, K is the stiffness matrix, and M is the mass matrix. The formula can be directly used to calculate the natural frequency of the plates under free vibration.

2.2. Material

The model developed for this study, as shown in Fig. 1, is inspired by the elastic composite stacking sequence $[0/90]_{2s}$ considered by Sinha et al. [34]. The composite laminate square model is modified by introducing a group of eccentric cutouts. The sizes and number of cutouts, in addition to the dimensions and the modular ratios of composite plates, are all considered parameters. The properties of the material used in this study are listed in Table 1. The position of the cutouts according to the x-axis is considered $e_x/a = 0.125$ for all the scenarios.

Table 1 Mechanical properties of the material used in this study

Material	Glass fiber-reinforced polymer (GFRP) [34]
E1 (N/m^2)	16.07×10^9
E2 (N/m^2)	16.07×10^9
G1 (N/m^2)	2.81×10^9
v_{12}	0.25
Density ρ (kg/m^3)	1664
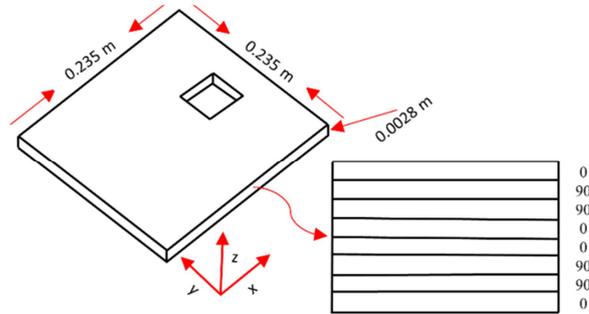
Fig. 1 Laminated composite plate with dimensions

2.3. FE simulations

The simulations are undertaken using ABAQUS. To determine the natural frequency, the FE method analysis is performed in a CFFF configuration on several specimens of laminated plates with cutouts. The size, number, length, thickness, and modular ratios of specimens are all different from one another.

2.4. Convergence

As a starting point, the simulation convergence with respect to different mesh sizes is verified by calculating the natural frequency for two mode shapes under boundary conditions (CFFF), as shown in Table 2. The result in terms of mesh density and mode shapes with respect to the chosen mesh size of 2 is represented in Fig. 2.

Table 2 Natural frequency in two mode shapes versus mesh size

Size	Natural frequency (Hz) for laminated composite plates with cutouts $[0/90]_{2s}$	
	First mode shape	Second mode shape
4	69.605	26.187
3.5	62.817	26.168
3	56.777	26.156
2.5	52.696	26.145
2	50.318	26.140
1.5	49.119	26.137

(a) Mesh density   (b) First mode shape of the plate   (c) Second mode shape of the plate
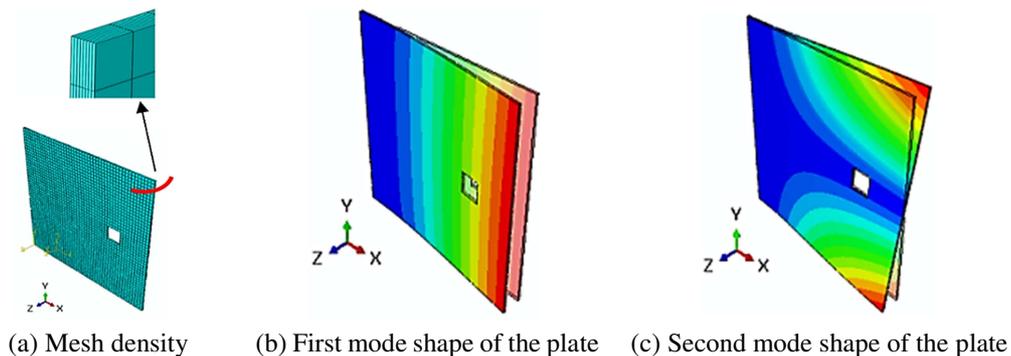
Fig. 2 Models of FE and mode shapes for laminated composite plates

2.5. Validation of the FE model

The numerical model is validated by comparing the natural frequency in Hz for CFFF plates that have the length of 0.235 m corresponding to the first mode predicted by this study to those provided by Sinha et al. [34] with the same parameters. As shown in Table 3, the parameters used in this study, especially the size and position of the cutouts, are same as those in the work of Sinha et al. [34].

Table 3 Comparison of the parameters used in this study and in the work of Sinha et al. [34]

Parameter		[34]		This study	Error
Position of cutout	Size of cutout	Experimental	FE model		
$e_x/a = e_y/b = 0.25$	d/a = 0.1	28	26.2	26.13	1.87
	d/a = 0.2	32	26.8	26.76	5.24
$e_x/a = 0.25$ and $e_y/b = 0$	d/a = 0.1	24	26.2	26.14	2.14
	d/a = 0.2	20	26.9	26.8	6.8

3. Development of the ANN Model

3.1. Artificial intelligence

Artificial intelligence is considered one of the most important and fastest developing fields in scientific research. This is due to the difficulty of finding solutions to many problems using conventional methods. ANNs are preferred among all the artificial intelligence types due to many characteristics: good data security throughout the whole network, the ability to function with less information, excellent fault tolerance, and the ability to train a machine using distributed memory and parallel processing capability.

3.2. Parameters and numerical data

The numerical model built in the previous section is used to acquire a dataset of 260 natural frequency values in each vibration mode. The feed-forward backpropagation network is implemented using MATLAB. The following entry variables are fed to the input layer: the size ratio (d/a), number of cutouts, modular ratio (E1/E2), length-to-width ratio (a/b), and thickness ratio (h/a) of the plates. The neurons are in the hidden layer, whereas the number of neurons is in the output layer. The number of neurons in the last layer is two, which represents the number of vibration modes. The input and hidden layers are served by tan-sigmoid transfer functions, while the output layer is served by a linear transfer function. Table 4 lists the parameters used to construct the dataset. The neural network is designed, developed, and deployed using the neural network toolbox in MATLAB.

Table 4 Simulation parameters

Size ratio (d/a)	0.1/0.2/0.3
Number of cutouts	1/2/4
Modular ratio (E1/E2)	1/2/3/4
Length-to-width ratio (a/b)	1/2/1.5
Thickness ratio (h/a)	0.012/0.018/0.024

In this study, a multilayer feed-forward neural network (FFNN) is used to determine the natural frequency. It consists of a single input layer, one or more hidden layers, and a single output layer. A neural network with a single hidden layer can handle the most complicated functions. The basic neural network model is denoted by:

$$\rho_j = \psi(\sum_i w_{ij} x_i + b_j) \tag{2}$$

Fig. 3 depicts a neuron's schematic structure.
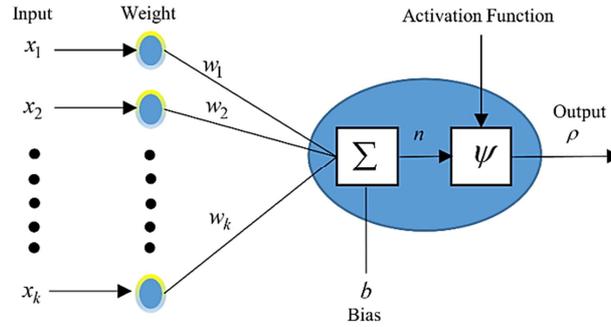
Fig. 3 Structure of a neuron

x_i denotes the set of inputs for every neuron. The collection of outputs for each neuron is indicated by ρ_j, while the bias set for each neuron is given by b_j. The weight coefficient w_{ji} is multiplied by every input and then summed with a bias b of neurons to generate the net input n, which may be written as:

$$n = \sum_{j=1}^{k} w_j x_j + b \tag{3}$$

The mathematical notation ji corresponds to the input I in neuron j. The net input n is then sent via an active function Ψ, which gives the neuron output ρ.

$$\rho = f(n) \tag{4}$$

The hyperbolic tangent sigmoid activation function is used in this investigation. The following formula may be used to express it:

$$\psi = \frac{e^n - e^{-n}}{e^n + e^{-n}} \tag{5}$$

As a result, FNNN with a single input layer and one hidden layer in Fig. 4 implements the equation below:

$$a^2 = \psi^2 \left[\sum_{i=1}^{N} w_{1i}^2 \psi^1 (\sum_{j=1}^{k} w_{ij}^1 x_j + b_i^1) + b^2 \right] \tag{6}$$

where a^2 signifies the entire network output. The activation functions of the hidden layer and output layer, accordingly, are represented by Ψ^1 and Ψ^2. k and N denote the number of inputs and neurons in the hidden layer, respectively. b^2 is the neuron's bias within the output layer. w_{1i} is the weight that connects the ith hidden layer resource to the output layer neuron. Fig. 4 depicts the conceptual architecture of the model.
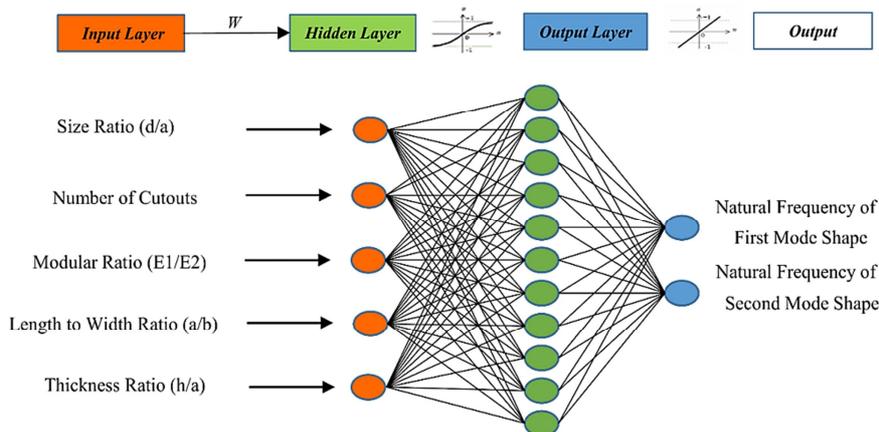
Fig. 4 Architecture of the proposed ANN predictive model

3.3. Levenberg-Marquardt algorithm

Training neural networks are basically nonlinear least-squares problems that can be solved using a class of nonlinear least-squares algorithms. Among them, the Levenberg-Marquardt approach is regarded as the most efficient algorithm for training ANNs. It is based on Newton's approach, which was developed for minimizing sums of squares error functions, such as the form below:

$$F(x) = \sum_{i=1}^{N} u_j^2(x) = \frac{1}{2} u^T(x)u(x) \tag{7}$$

where u_j is the error in the Nth pattern, u is the vector with elements u_j, and $x = (w_1, w_2, w_3, \ldots, w_N)^T$ contains all of the network's weights. The sum of squared errors function is denoted as:

$$F(w) = \frac{1}{2} u^T(w)u(w) \tag{8}$$

Newton's approach is used to maximize a performance index $F(w)$:

$$w_{N+1} = w_N - A_N^{-1} g_N \tag{9}$$

$$A_N \equiv \nabla^2 F(w)\big|_{w=w_N} \tag{10}$$

$$g_N \equiv \nabla F(w)\big|_{w=w_N} \tag{11}$$

where $\nabla^2 F(w)$ is the Hessian matrix and $\nabla F(w)$ is the gradient obtained as follows.

$$\nabla F(w) = 2J^T(w)u(w) \tag{12}$$

$$\nabla^2 F(w) = 2J^T(w)J(w) + 2S(w) \tag{13}$$

$J(w)$ represents the Jacobian matrix:

$$J(w) = \begin{bmatrix} \frac{\partial u_{11}}{\partial w_1} & \frac{\partial u_{11}}{\partial w_2} & \cdots & \frac{\partial u_{11}}{\partial w_N} \\ \frac{\partial u_{12}}{\partial w_1} & \frac{\partial u_{12}}{\partial w_2} & \cdots & \frac{\partial u_{12}}{\partial w_N} \\ \vdots & \vdots & \ddots & \cdots \\ \frac{\partial u_{1M}}{\partial w_1} & \frac{\partial u_{1M}}{\partial w_2} & \cdots & \frac{\partial u_{1M}}{\partial w_N} \\ \cdots & \cdots & \ddots & \cdots \\ \frac{\partial u_{P1}}{\partial w_1} & \frac{\partial u_{P1}}{\partial w_2} & \cdots & \frac{\partial u_{P1}}{\partial w_N} \\ \frac{\partial u_{P2}}{\partial w_1} & \frac{\partial u_{P2}}{\partial w_2} & \cdots & \frac{\partial u_{P2}}{\partial w_N} \\ \cdots & \cdots & \ddots & \cdots \\ \frac{\partial u_{PM}}{\partial w_1} & \frac{\partial u_{PM}}{\partial w_2} & \cdots & \frac{\partial u_{PM}}{\partial w_N} \end{bmatrix} \tag{14}$$

where P stands for the number of training patterns and M defines the number of output patterns.

$$S(w) = \sum_{i=1}^{N} v_i(w)\nabla^2 v_i(w) \tag{15}$$

The Hessian matrix can be estimated as follows if $S(x)$ is considered to be small.

$$\nabla^2 F(w) = 2J^T(w)J(w) \tag{16}$$

Substituting Eqs. (11) and (15) into Eq. (8), the Gauss-Newton method can be obtained as:

$$\Delta w_k = -\left[J^T(w_k)J(w_k) \right]^{-1} J^T(w_k)u(w_k) \tag{17}$$

The Gauss-Newton method has a problem in that the matrix may not be invertible. This may be avoided by making the following changes to approximate the Hessian matrix:

$$G = H + \mu I \tag{18}$$

As a result, the Levenberg-Marquardt algorithm is expressed as:

$$\Delta w_k = -\left[J^T(w_k)J(w_k) + \mu_k I \right]^{-1} J^T(w_k)u(w_k) \tag{19}$$

where I is the identity matrix and the amount μ is referred to as the learning parameter in neural computing. The learning parameter is reduced as the iterative procedure nears its end. The Levenberg-Marquardt algorithm is used in this study, as in the work of Hagan et al. [35] and Lv et al. [36].

3.4. Validation of the ANN model

The development of an ANN model begins by feeding all the captured data as "given" inputs and "desired" outputs. The data is then divided into three sets: training, validation, and test sets, with the proportions 70%, 15%, and 15%, respectively. The regression coefficient (R) and the mean squared error (MSE) are used to validate the generated model's appropriateness. For all the data, R and MSE are 0.99995 and 0.143, respectively. As the optimum network, a hidden layer with eleven neurons is chosen to minimize MSE. Table 5 shows the findings. The results predicted by the ANN model are extremely close to the numerical ones. This proves that ANN can successfully forecast the natural frequency of laminated composite plates with a cutout.

To examine the performance of the proposed model even more deeply, a regression analysis of the predicted as well as the numerical results is illustrated in Fig. 5. The regression coefficients (R) are calculated to determine the correlation between the ANN predicted values and those obtained from the FE model, as shown in Fig. 6. They are divided into three sets: the training, validation, and test data sets. The value of the coefficient R is between zero and one. The degree of correlation increases as R tends toward one. The ANN structure (5-11-2) is the best one in Table 5.

The convergence tests of ANN results are done with MSE and the regression correlation coefficient (R):

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y})^2 \tag{20}$$

$$R = \left\{ 1 - \left[\sum_{i=1}^{n} (y_i - \hat{y})^2 \Big/ \sum_{i=1}^{n} (y_i - \overline{y})_i^2 \right] \right\}^{\frac{1}{2}} \tag{21}$$

where y is the actual value, $\hat{y}$ is the predicted value of y, and $\overline{y}$ is the mean value of y.
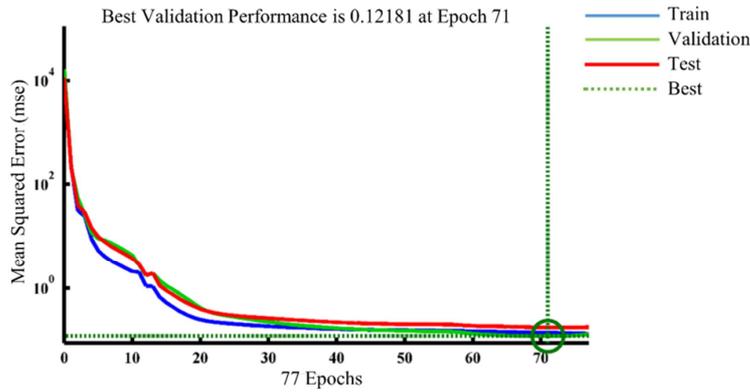
Fig. 5 Performance of ANN training
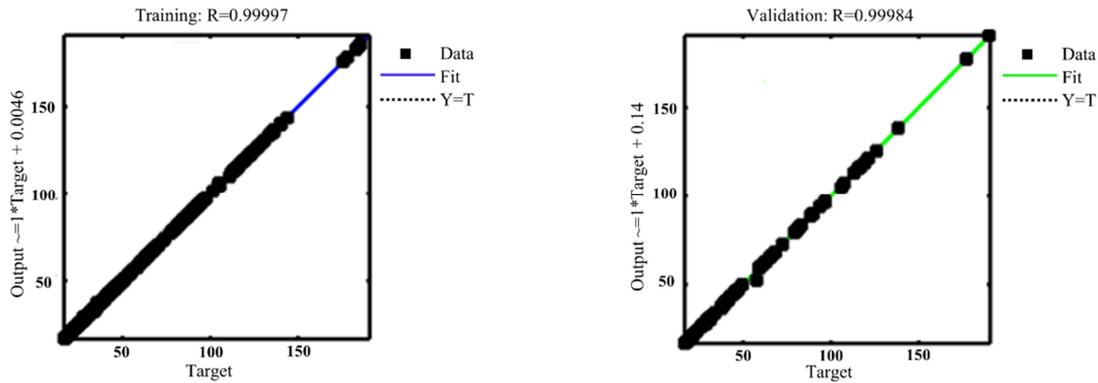
(a)Training results of a neural network regression

(b) Validation results of a neural network regression
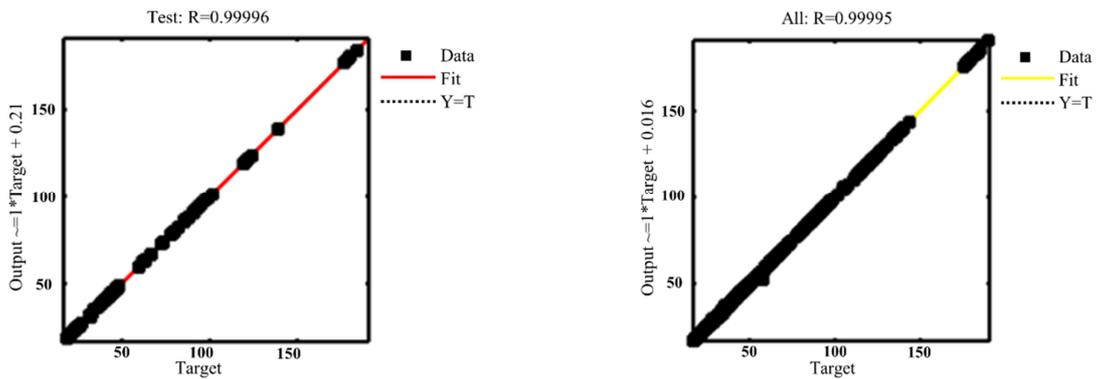
(c) Test results of a neural network regression

(d) Neural network regression in all data sets

Fig. 6 Correlation between the values predicted by the ANN model with structure (5-11-2) and by the FE model

Table 5 MSE and regression coefficient according to the number of hidden layer neurons

ANN structure	The performance of training	Training: R	Validation: R	Test: R	Mean squared error (MSE)
5-1-2	40.2782	0.98244	0.98178	0.98124	49.157
5-2-2	11.8044	0.99125	0.99529	0.98987	24.133
5-3-2	11.8873	0.99421	0.99661	0.99225	16.094
5-4-2	12.9589	0.99613	0.99452	0.99575	11..775
5-5-2	6.0636	0.99802	0.99788	0.99750	6.028
5-6-2	8.8178	0.99376	0.99785	0.99353	15.627
5-7-2	9.9689	0.99729	0.99736	0.99609	8.449
5-8-2	0.8028	0.99974	0.99968	0.9990	7.940
5-9-2	1.6912	0.99951	0.99953	0.99905	1.588
5-10-2	0.5544	0.99985	0.99980	0.99988	0.433
5-11-2	0.1218	0.99995	0.99995	0.99995	0.12
5-12-2	0.5212	0.99989	0.99980	0.99967	0.424
5-13-2	0.3645	0.99996	0.99987	0.99979	0.294
5-14-2	0.3186	0.99994	0.99990	0.99982	0.244
5-15-2	0.5560	0.99993	0.99982	0.99981	0.298

3.5. Comparison between the numerical and predicted natural frequency for all data sets

Figs. 7(a)-(b) provide a comparison between the numerical and the ANN-predicted natural frequency of the first mode shapes. In terms of absolute error (AE), the highest values are 2.94% and 1.58% for each mode shape, while the lowest values are 0.03% and 0%. Overall, the error is close to zero, which proves the validity and the accuracy of the ANN model. It is noteworthy that AE is determined using the following relation to the positive value.

$$AE = y - \hat{y} \tag{22}$$

where y is the actual value, and $\hat{y}$ is the predicted value of y.

The proposed ANN model is then used to investigate the effect of several variables on the natural frequency as a function of both geometrical and mechanical parameters. Only one parameter is changed at a time, while all others are maintained constant. The sensitivity of the composite plate characteristics with respect to the cutout parameters is investigated in the following sections.
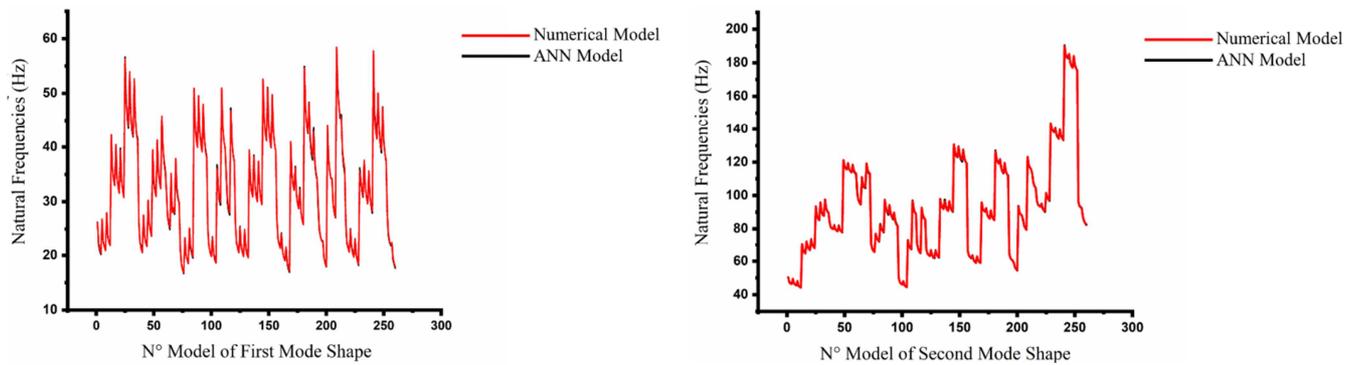
(a) The values of natural frequency in the first mode shape   (b) The values of natural frequency in the second mode shape

Fig. 7 Comparison of the natural frequency values between the numerical model and
the ANN model with structure (5-11-2) in two mode shapes

4. Analysis of Parameters' Effect

4.1. Effect of the modular ratio (*E1/E2*)

The influence of the modular ratio on the natural frequency of the CFFF laminate composite plate $[0/90]_{2s}$ is studied with the following characteristics: length-to-width ratio = 1, thickness ratio = 0.012, and the square cutout with a size ratio d/a = 0.1. The change in natural frequency of the two vibration modes is predicted for the following combinations: E1/E2 = 1, 2, 3, and 4.

Table 6 shows that, for the two mode shapes, the value of the frequency decreases as the modular ratio increases. It is worth noticing that the natural frequency corresponding to the modular ratio of 2 and the ones corresponding to 3 and 4 are very close. The lowest natural frequencies are 20.36 Hz and 46.37 Hz for each mode, and the highest ones are 26.140 Hz and 50.316 Hz.

Table 6 Natural frequencies of the laminated square plate with different parameters (the modular ratio, cutout size ratio, and number of cutouts)

-	Modular ratio (E1/E2)				Cutout size ratio (d/a)			Number of cutouts		
	1	2	3	4	0.1	0.2	0.3	1	2	4
Natural frequency of the first mode vibration	26.14	22.41	21.06	20.36	26.14	26.80	27.98	36.06	37.92	39.46
Natural frequency of the second mode vibration	50.31	47.78	46.66	46.37	50.31	49.49	48.16	73.16	73.21	73.66

4.2. *Effect of the cutout size*

Table 6 also shows the variation in the first two natural frequencies with respect to three different cutout size ratios: $d/a =$ 0.1, 0.2, and 0.3 in the same laminated plate having length-to-width ratio $a/b = 1$ and thickness ratio $h/b = 0.012$. It is obvious that the size of the cutout affects the natural frequencies of the composite plate.

According to the results, the natural frequency of the specimen and the size of the cutout are proportional. For the first two modes, when the size of the cutout d/a changes from 0.1 to 0.2, 0.1 to 0.3, and 0.2 to 0.3, the natural frequency increases by 2.54%, 1.64%, and 2.23%, then by 7.03%, 4.26%, and 5.53%.

4.3. *Effect of the cutout number*

In this part, the effect of the number of cutouts on natural frequency is investigated. The laminated square plate is composed of eight elastic layers $[0/90]_{2S}$ with CFFF support in the borders. The natural frequency of the first two mode shapes of the composite plate is determined using a frequency response study. The square laminate utilized has a length-to-width ratio of $a/b = 1$ and a thickness ratio of $h/b = 0.01$, whereas the cutout size ratio equals $d/a = 0.1$ in all three configurations. The results are displayed in Table 6.

For both the vibration modes, the number of cutouts has an inverse effect on the natural frequency of the laminated composite plates. According to the results, as the number of cutouts increases, the specimen's natural frequency decreases by a modest amount. The change does not exceed 0.7% for the second vibration mode.

5. Conclusions

In this study, a neural-network-based approach is proposed to assess changes in the geometrical and mechanical properties of composite plates through the prediction of natural frequency. The main idea is to simulate the composite model numerically with the FE method and use its output to construct and train a successful ANN predictive model. The model uses natural frequency as an indicator. After validation, the ANN model is used to identify the changes in structures by the prediction of natural frequency. The effects of the main geometrical and mechanical characteristics (e.g., the cutout size ratio (d/a), the number of cutouts, and the modular ratio $(E1/E2)$) on the natural frequency were investigated. The findings of this study can be summarized as follows:

(1) The constructed ANN model agrees with the FE model, indicated by a mean squared error near zero. The greatest AE between the numerical model and the ANN forecasting model was found to be 4.92% and 1.75% for the first and the second vibration modes, respectively.

(2) The natural frequency of the laminate plates is inversely proportional to the E1/E2 ratio

(3) The natural frequency is inversely proportional to the number of cutouts.

(4) The natural frequency increases with the increase in the size of the cutouts.

Conflicts of Interest

The authors declare no conflict of interest.

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