Dynamic Poisson's Ratio and Modulus of Elasticity of Pozzolana Portland Cement Concrete

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Abstract

An experimental investigation was carried out to determine the dynamic Poisson's ratio and modulus of elasticity of pozzolana Portland cement concrete. Large numbers of concrete cubes were prepared and tested in the laboratory for investigation. The destructive and non-destructive tests were conducted on concrete cubes at different ages. The destructive test was conducted to obtain the compressive strength of concrete cubes. Ultrasonic pulse velocity of cube specimens was derived according to IS 13311 (Part 1) and the transit time of longitudinal and shear waves transmission was recorded. The recorded values are used to determine the dynamic values of Poisson's ratio and elastic modulus of concretes. Based on experimentally obtained data and analysis, a few relationships are proposed to correlate the water/cement ratio, Poisson's ratio, elastic moduli and compressive strength of concretes. Several interesting findings are observed from the correlations of the coefficients while analyzed by regression analysis. This study helps to determine the static properties of concrete from the dynamic ones.

Keywords: pozzolana Portland cement, Poisson's ratio, modulus of elasticity, ultrasonic pulse velocity, mix proportion

1. Introduction

According to IS 456 clause 6.1 [1], the concretes are classified into three groups, i.e., low, medium, and high strength concretes. Today, low and medium strength concretes are the most commonly used concrete in the construction of civil engineering structures. The Poisson's ratio and elastic modulus are the significant parameters used in the analysis and design of concrete structures. The Poisson's ratio of a material influences the speed of propagation and reflection of stress waves. How a concrete specimen bulges when subjected to compressive load and the ability of a material to be strained can be easily known with the value of Poisson's ratio. The static modulus of elasticity can be estimated from the compressive strength according to the standard design code rather than on direct measurement. However, this practice underestimates the demand of higher compressive strength to achieve the desired value that is actually required in the structural design. Hence, the dynamic properties of concrete should be ensured in addition to the concrete compressive strength. Although, the dynamic modulus of elasticity can be determined by assuming the value of Poisson's ratio but 30% error may be introduced in the calculated values of dynamic modulus [2]. For an existing structure, dynamic Poisson's ratio, and modulus of elasticity can be determined by the non-destructive testing method. The dynamic values obtained by NDT cannot match with the static values, but their relationships can be established. In order to establish their relationships, regression technique may be considered to correlate the coefficients.

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The properties determined by NDT are the density, moduli of elasticity, Poisson's ratio, strength, surface hardness, and surface absorption of concrete. In addition to that, the NDT has the ability to detect the voids and cracks in concrete. One of the most effective NDT is the UPV test, which is applied to assess the structural integrity without causing any damage. In UPV test method [3], two transducers are used to produce and receive the ultrasonic pulse (electrical signal). While the receiving transducer is positioned exactly on the opposite face of the test sample (direct method or cross probing), and qualitative results are obtained. However, in many situations, two opposite faces of the structural member may not be accessible for the test. In such cases, a semi-direct method can be adopted satisfactorily if the path length is not too large. The transducers can also be placed on the same side of the concrete members which is known as a surface probing method. However, the surface probing technique is not as efficient as that of cross-probing, because the test results are influenced by the surface layers of concrete, which is having the different properties from that of concrete inside. The indirect velocity is always lower than the direct velocity and the difference may vary depending on the quality of the concrete under test. Carette and Staquet [4] observed that the shear wave velocity is more sensitive to the setting process of cement-based materials than the longitudinal wave velocity. The quality of concrete can be characterized in terms of pulse wave velocity. However, the pulse wave velocity depends on the elastic properties and the density of medium [5], therefore, UPV test can be adopted to determine the properties of concrete. The dynamic Poisson's ratio of concrete can be calculated using the following equation [3];

$$\mu_d = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - 2V_s^2)} \quad (1)$$

The dynamic modulus of elasticity of concrete can be calculated using the following equation, if the density and the Poisson's ratio of concrete are known [3];

$$E_d = \rho V_p^2 \frac{(1+\mu_d)(1-2\mu_d)}{(1-\mu_d)} \quad (2)$$

For a linear elastic and isotropic material, Poisson’s ratio is constant but in concrete, Poisson's ratio is influenced by some specific conditions. Lydon and Balendran [6] observed that the Poisson's ratio of concrete lies in the range of 0.15 to 0.22 when determined from strain measurements under the compressive load. The similar range of Poisson's ratio was obtained under the tensile load. The dynamic Poisson's ratio can be determined experimentally and the physical situation of such test is completely different from that of static loading. The value of dynamic Poisson's ratio is always higher than the static Poisson's ratio; an average value is about 0.24 [7]. Leslie and Cheesman [8] used the value of Poisson's ratio of 0.24 for the concretes having a density in excess of 2240 kg/m^3 and found good results.

In general, the modulus of elasticity increases with the compressive strength of concrete. However, the modulus of elasticity of aggregates and hydrated cement paste affect the modulus of elasticity of concretes. The static modulus of elasticity of concrete, recommended by IS 456 [1], can be assumed as

$$E_c = 5.0\sqrt{f_{ck}} \quad (3)$$

Where E_c is in GPa and f_{ck} is in MPa. The actual measured values may differ by ±20% from the values obtained from Eq. (3). The relationship for the secant modulus of elasticity, recommended by ACI 318 [9] was

$$E_c = 4.73\sqrt{f_{ck}} \quad (4)$$

In the range of compressive strength between 80 to 140 MPa, Kakizaki et al. [10] found that the modulus of elasticity is approximately related to the compressive strength by

$$E_c = 3.65\sqrt{f_{ck}} \tag{5}$$

The density of concrete is increased when the density of aggregates increases. The density of lightweight aggregates is lower than the hydrated cement paste, which influences the modulus of elasticity of concrete. However, the normal weight aggregates have a higher density than the hydrated cement paste and as a result, higher modulus of elasticity of concrete is obtained for a given compressive strength. When the density of concrete is between 2320 and 2480 kg/m^3, the modulus of elasticity was given by ACI 318 [9] as

$$E_c = 43\rho^{1.5}\sqrt{f_{ck}} \times 10^{-6} \tag{6}$$

Where ρ is in kg/m^3. The ratio of static to dynamic modulus is always smaller than unity as the dynamic value is unaffected by creep. The ratio is increased with the age and is higher for a higher strength of concrete [11]. Various relationships were proposed by different researchers. The simplest of these, proposed by Lydon and Balendran [6] was

$$E_c = 0.83E_d \tag{7}$$

Where E_d is in GPa. The British testing standard [12] provided the following empirical equation;

$$E_c = 1.25E_d - 19 \tag{8}$$

However, Eq. (8) does not apply for the concretes that contain more than 500 kg/m^3 of cement. Swamy and Bandyopadhyay [13] suggested the following expression;

$$E_c = 1.04E_d - 4.1 \tag{9}$$

For normal density concretes, Popovics [14] suggested the following relationship in terms of density of concrete;

$$E_c = 446.09E_d^{1.4}\rho^{-1} \tag{10}$$

Lu et al. [15] applied the impact-echo method to determine the dynamic modulus of elasticity of concrete. Authors found that the longitudinal wave velocity on standard specimens matches well with the theoretical one-dimensional longitudinal wave velocity. The following relationship was proposed by authors' in estimating the dynamic modulus of elasticity from the static modulus of elasticity.

$$E_d = 1.15E_c \tag{11}$$

Nematzadeh and Naghipour [16] provided an interesting solution to enhance the compressive strength and elastic modulus of concrete. Krizova and Hela [17] described the influence of various factors on elasticity modulus of concrete. Authors presented that the results of the elastic modulus of concrete are mainly influenced by water coefficient and the size of the maximum fraction of the aggregate. Jurowski and Grzeszczyk [18] presented a dynamic testing method for determining the Young's modulus of self-compacting concrete. Kulkarni et al. [19] investigated experimentally the elastic modulus of recycled aggregate concrete. Authors found that the elastic modulus decreases with the increase in percentage replacement of recycled aggregate. Lee et al. [20] evaluated the dynamic elastic modulus of three different concretes with target compressive strengths of 30, 35 and 40 MPa using the shear-wave velocity measurements.

The aim of the study is to provide some relevant data on dynamic Poisson's ratio and modulus of elasticity; in which the properties of low and medium strength concretes are the main subject of investigation. In order to compare the low strength with the medium strength concrete, several testing were performed in the laboratory, conforming to the relevant Indian standards. Since the low and medium strength concretes produced are based on the same materials and grading; they are easily comparable. UPV of the concretes were experimentally obtained and the dynamic values of Poisson's ratio and modulus of elasticity of concretes are calculated. The results are presented with the correlation of the predicted models by regression analysis.

2. Experimental Study

2.1. Material properties

Concrete is produced by the homogenous mixing of suitable proportions of cement, water, and aggregates. The physical properties of the materials, which were obtained experimentally in the laboratory, are described in the following sections.

2.1.1. Cement

Pozzolana Portland cement is normally used in concrete technology nowadays and it is used in various quantities to obtain a wider range of mixtures of low and medium strength concretes. PPC, conforming to IS 1489 [21] was used in this investigation in preparing the concrete mixtures. The tests were performed in the laboratory to determine the physical and mechanical properties of the cement and the obtained values are presented in Table 1.

Table 1 Properties of PPC

Properties	Value obtained
Normal consistency	31.6%
Initial setting time	85 minutes
Final setting time	225 minutes
Specific gravity	2.92
3-days compressive strength of 70.6 mm cube sample	18.05 MPa
7-days compressive strength of 70.6 mm cube sample	25.08 MPa
28-days compressive strength of 70.6 mm cube sample	38.12 MPa
Fineness (i.e. by using 90µ sieve)	0.67%
Soundness (i.e. by using Le-Chatelier’s apparatus)	2.5 mm

2.1.2. Aggregates

Locally available natural sand, conforming to IS 383 [22] was used as fine aggregate (FA). Local aggregates of size 20 mm and 10 mm conforming to IS 383 were used as coarse aggregates (CA). Well graded coarse aggregates 20 mm and 10 mm were combined in the proportion as 60:40 by mass. The physical properties of aggregates, required for mix proportioning, are presented in Table 2.

Table 2 Properties of aggregates

Properties	Value obtained
Specific gravity of FA	2.72
Fineness modulus of FA	2.56
Moisture present in FA	2.16%
Water absorption of FA	1.05%
Grading zone of FA	III
Specific gravity of 10 mm CA	2.67
Fineness modulus of 10 mm CA	6.34
Water absorption of 10 mm CA	0.51%
Specific gravity of 20 mm CA	2.62
Fineness modulus of 20 mm CA	7.31
Water absorption of 20 mm CA	0.43%

2.1.3. Water

Ordinary potable water available in the laboratory was used in preparing the mixtures.

2.2. Mix proportioning

In general, mix proportioning is carried out for a particular compressive strength requirement ascertaining that the fresh concrete is having the adequate workability without any segregation and bleeding. Mix design is a process of selecting and proportioning the ingredients in right proportions to produce good quality concrete as economically as possible to achieve the required minimum strength and durability. The physical properties of materials (i.e., Tables 1 and 2) were used in the design of mix proportioning to produce different types of concretes, conforming to IS 456 [1] and IS 10262 [23].

Table 3 Mix proportion of PPC concretes

Type of Concrete	Grade of Concrete	Slump (mm)	W/C ratio	Cement (kg/m^3)	FA (kg/m^3)	10 mm CA (kg/m^3)	20 mm CA (kg/m^3)	Water (kg/m^3)
Low strength concrete	M 15	25-50	0.60	255	655.35	516.12	774.18	153.00
		50-75	0.58	280	658.00	500.64	750.96	162.40
		75-100	0.55	318	674.16	475.59	713.59	174.90
	M 20	25-50	0.51	346	671.24	476.10	714.14	176.46
		50-75	0.48	388	628.56	473.36	710.04	186.24
		75-100	0.47	408	624.24	463.49	695.23	191.76
Medium strength concrete	M 25	25-50	0.49	384	610.56	482.30	723.46	188.16
		50-75	0.47	410	582.20	480.52	720.78	192.70
		75-100	0.46	432	578.88	471.74	707.62	198.72
	M 30	25-50	0.45	410	578.10	490.36	735.54	184.50
		50-75	0.44	438	573.78	478.30	717.44	192.72
		75-100	0.43	476	547.40	468.38	702.58	204.68
	M 35	25-50	0.44	465	567.30	466.86	700.29	204.60
		50-75	0.42	488	550.31	465.57	698.36	204.54
		75-100	0.41	496	545.60	464.26	696.38	203.36

In this study, five different grades (i.e. M15-M35) of low and medium strength concretes were considered for the investigation. In each grade, three different types of mix were prepared by varying the water/cement ratios. The maximum particle size of aggregate was kept constant as 20 mm. The quantity of water, cement, fine, and coarse aggregates required to make one cubic meter of concrete was calculated and are presented in Table 3 for the different grades of concretes. The produced concretes were classified into two groups [1], i.e., low strength (M15 and M20) and medium strength (M25, M30, and M35) concretes. According to IS 456 clause 7.1, the degree of workability is classified into several groups. Three different ranges of workability (i.e. 25-50 mm, 50-75 mm, and 75-100 mm) were obtained in fresh concrete while preparing the mixes. Abram's cone apparatus was used to measure the workability (slump value) of different mixes. The vibration table was used for compaction of the concrete cubes. 18 numbers of cube samples in each grade and in total, 90 numbers of cube samples were prepared in the laboratory for investigation. The plain cement concrete cubes of 150×150×150 mm size (Fig. 1) were demolded after 24 hours of casting and put in a tap-water chamber for 28-days curing (Fig. 2). Compression test was conducted on a 2000 kN capacity compression testing machine and the load was applied as per IS 14858 [24].

Fig. 1 Concrete cube samples


Fig. 2 Curing tank for cube samples

2.3. UPV test

PUNDIT (Portable Ultrasonic Nondestructive Digital Indicating Tester) equipment with flat transducers was used for UPV test in which one transducer was held in contact with one face of the concrete cube to produce the ultrasonic pulse. The electrical signal was received by another transducer, which was held in contact with the other face of the same concrete cube. A thin film of paraffin wax was used between the molded faces of the cube specimens and the transducers to obtain the qualitative results. The best combination of results were obtained while the receiving transducer was positioned exactly on the opposite face of the test sample. The shear wave velocity was measured using the process similar to the longitudinal wave velocity measurement.

Five different grades of concrete were taken in order to cover the different types of concretes, generally used. The UPV and the relevant crushing strength tests were conducted on the concrete cubes to check their acceptance. One of the important factors affecting the modulus of elasticity is the crushing strength of concrete. Longitudinal (Fig. 3) and shear wave (Fig. 4) velocity of the controlled samples were recorded while testing of compressive strength. Concrete cubes were tested at the age of 7-days and 28-days; however, the results at the curing age of 28-days are interpreted (i.e. the results of 45 numbers of cube samples) herein for the sake of conciseness of the results.

Fig. 3 Longitudinal wave velocity measurement

Fig. 4 Shear wave velocity measurement

3. Results and Discussion

The recorded results in each grade of concretes are tabulated and the dynamic Poisson's ratio and modulus of elasticity are calculated using Eqs. (1) and (2). Table 4 presents the test results of both low and medium strength concretes. Regression analysis is performed to study the correlations among the observed data and to establish the mathematical relationships between the parameters. The nature of each relationship is proposed from the level of accuracy of statistical measure of fitted regression line.

The dynamic Poisson’s ratio is initially high for the lower grade of concrete and decreases with the higher grade of concrete. The Poisson's ratio is increased with the increase in water/cement ratio. The dynamic modulus of elasticity increases with the grade of concrete. However, Poisson's ratio and modulus of elasticity of the aggregates influence the Poisson's ratio and modulus of elasticity of the resulting concrete [25]. According to the fib model code [26], the elastic deformation of concrete mostly depends on the type of aggregates. The modulus of elasticity of concrete is increased by 20% with the use of quartzite aggregates and it is decreased by 30% with other types of aggregates. Locally available aggregates nearby Allahabad are limestone, granite, basalt, and dolerite. In this study, the limestone aggregates were used in the mixes. The variation of dynamic Poisson's ratio with the dynamic modulus of elasticity is illustrated in Fig. 5. The Poisson's ratio decreases with the increase in elastic modulus, but the results are found to be too scattered and not practicable to establish any relationship. However, a new formulation for correlating the elastic modulus and Poisson's ratio is proposed, shown in Fig. 5. Even though, these parameters are theoretically correlated with each other from Eqs. (1) and (2). But the proposed formulation may be useful to calculate one of the parameters by knowing the other one. Even so, for a wide range of mixes, Simmons [27] found considerable scattered values between the Poisson's ratio and the dynamic modulus. The author showed the linear relationship between the Poisson's ratio and the dynamic modulus. However, for one mix proportion only, Krenchel [28] reported the values of Poisson's ratio of 0.15 to 0.18 for weak concretes and of 0.17 to 0.25 for strong concretes. The discrepancies found in the results may be due to the anisotropy of concrete as the heterogeneity of cementitious materials makes the anisotropic conditions. In good quality concretes, such variations may be neglected. Eqs. (1) and (2) used to calculate the dynamic Poisson’s ratio and elastic modulus are purely based on the assumption of concrete being a homogeneous, isotropic and elastic material.

Table 4 Test results of PPC concretes

Grade of concrete	W/C ratio	Slump (mm)	Mean compressive strength, f_{cm} (MPa)	Unit weight of concrete, ρ (kg/m^3)	UPV (km/s)		Dynamic Poisson’s ratio, μ_d	Dynamic modulus of elasticity, E_d (GPa)
					Longitudinal pulse velocity, V_p	Shear pulse velocity, V_s		
M15	0.60	25-50	20.88	2408.40	3.59	1.96	0.288	23.83
			22.00	2412.20	3.63	1.97	0.291	24.18
			21.33	2406.70	3.61	1.96	0.291	23.87
	0.58	50-75	22.44	2414.20	3.64	1.98	0.290	24.42
			21.77	2416.80	3.61	1.97	0.288	24.16
			21.44	2421.10	3.62	1.96	0.293	24.05
	0.55	75-100	22.22	2405.80	3.63	1.98	0.288	24.30
			21.55	2408.40	3.64	1.96	0.296	23.98
			21.77	2416.20	3.63	1.96	0.294	24.03
M20	0.51	25-50	28.00	2429.41	3.86	2.15	0.275	28.64
			26.66	2440.03	3.83	2.12	0.279	28.06
			27.77	2456.72	3.84	2.13	0.278	28.48
	0.48	50-75	28.44	2425.01	3.89	2.15	0.280	28.70
			27.33	2436.80	3.84	2.13	0.278	28.25
			28.22	2418.03	3.88	2.15	0.278	28.58
	0.47	75-100	27.11	2432.81	3.84	2.13	0.278	28.21
			28.88	2446.42	3.87	2.15	0.277	28.88
			27.77	2428.21	3.85	2.14	0.276	28.39
M25	0.49	25-50	31.77	2435.63	4.03	2.33	0.249	33.03
			31.33	2445.41	4.02	2.32	0.250	32.91
			30.22	2454.80	3.99	2.31	0.248	32.69
	0.47	50-75	33.55	2434.61	4.07	2.36	0.247	33.81
			32.00	2448.53	4.03	2.35	0.242	33.60
			32.88	2430.42	4.07	2.37	0.243	33.95
	0.46	75-100	30.66	2459.30	3.99	2.31	0.248	32.75
			31.11	2442.41	4.00	2.34	0.240	33.16
			34.44	2436.22	4.09	2.37	0.247	34.13
M30	0.45	25-50	38.22	2437.83	4.09	2.42	0.231	35.14
			39.77	2456.81	4.14	2.42	0.240	35.70
			39.11	2445.70	4.10	2.41	0.236	35.12
	0.44	50-75	40.66	2435.71	4.16	2.43	0.241	35.70
			41.33	2464.73	4.13	2.44	0.232	36.15
			39.77	2446.44	4.14	2.42	0.240	35.55
	0.43	75-100	42.66	2454.51	4.19	2.45	0.240	36.55
			41.77	2463.70	4.13	2.44	0.232	36.14
			42.22	2446.81	4.15	2.45	0.233	36.20
M35	0.44	25-50	45.11	2456.68	4.29	2.52	0.237	38.58
			44.00	2465.18	4.26	2.50	0.237	38.13
			44.88	2462.22	4.27	2.51	0.236	38.35
	0.42	50-75	45.11	2473.26	4.26	2.51	0.234	38.46
			43.77	2491.85	4.23	2.49	0.235	38.16
			43.55	2468.70	4.24	2.50	0.234	38.07
	0.41	75-100	43.11	2490.74	4.21	2.48	0.234	37.82
			44.88	2486.67	4.24	2.50	0.234	38.34
			44.44	2464.81	4.26	2.51	0.234	38.33

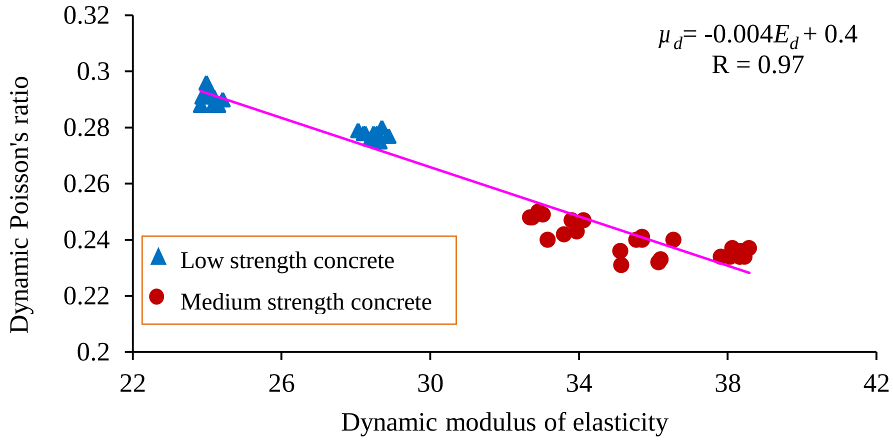

Fig. 5 Relationship between the dynamic modulus of elasticity and the dynamic Poisson's ratio

Fig. 6 shows the variation of the dynamic modulus of elasticity with the compressive strength. The variation of the dynamic Poisson's ratio with the compressive strength is shown in Fig. 7. It is found that the elastic modulus increases with the compressive strength but the Poisson's ratio tends to decrease with the increase in compressive strength. The Poisson's ratio decreases with the increase in water/cement ratio (Table 4), but they appear to be different relationships for different mixes. The similar observations were found in the literature [2]. The estimation models for different grades of concretes are proposed herein to determine the dynamic modulus of elasticity and Poisson's ratio. In each grade of concretes, the relationships between the mean compressive strength and the dynamic properties are obtained as

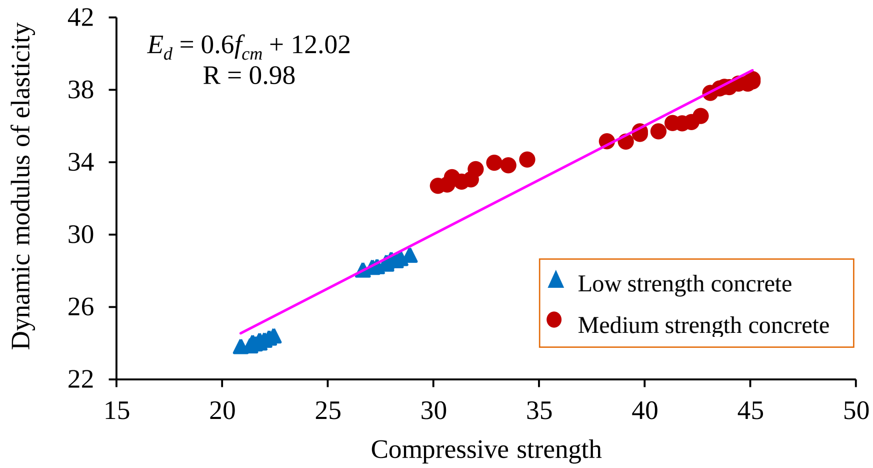

Fig. 6 Relationship between the compressive strength and the dynamic modulus of elasticity

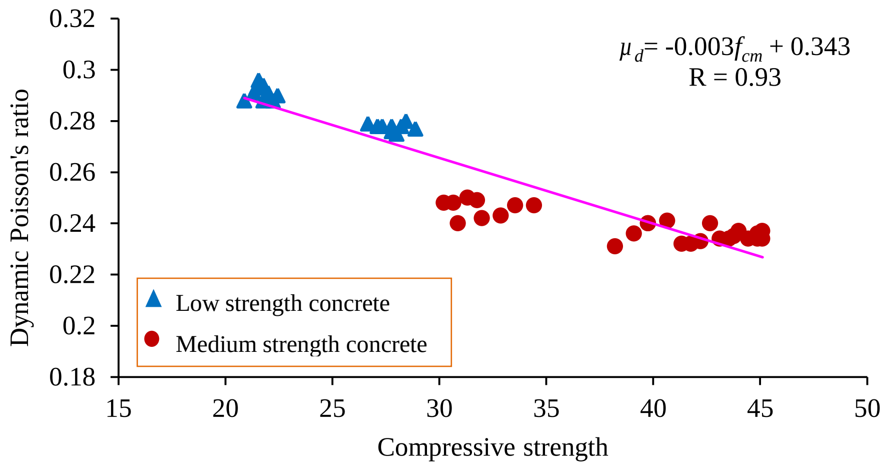

Fig. 7 Relationship between the compressive strength and the dynamic Poisson's ratio

Where E_d is in GPa and f_{cm} is in MPa. The relationship between the parameters [i.e., Eqs. (12)-(16)] is found to be unlike from each other. One may say that, instead of using a single expression (i.e., equations shown in Figs. 6 and 7), separate expression for different grades of concretes gives better result in estimating the dynamic properties of concretes. As seen in figures, the modulus of elasticity increases with compressive strength. Neville [29] reported that whenever the compressive strength increases, both the cement paste matrix and interface zones become denser and stronger, and as a result, higher modulus of elasticity is obtained. The higher cement content of the medium strength concretes results in higher rate of gain of the modulus of elasticity than the low strength concretes. The low strength concretes normally have high water/cement ratio and low cement content. In such mixes, the aggregate-cement interface zone is weaker than the medium strength concretes. The aggregates can easily be dissociated in the concretes having low compressive strength. For high water/cement ratio, bleeding will be more frequent, which may be another reason behind the low modulus of elasticity.

$$\left.\begin{aligned} E_d &= 3.59\sqrt{f_{cm}} + 7.33 \\ \mu_d &= 0.225\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \end{aligned}\right\} for\ M15 \tag{12}$$

$$\left.\begin{aligned} E_d &= 3.95\sqrt{f_{cm}} + 7.63 \\ \mu_d &= 0.197\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \end{aligned}\right\} for\ M20 \tag{13}$$

$$\left.\begin{aligned} E_d &= 4.18\sqrt{f_{cm}} + 9.67 \\ \mu_d &= 0.167\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \end{aligned}\right\} for\ M25 \tag{14}$$

$$\left.\begin{aligned} E_d &= 4.03\sqrt{f_{cm}} + 10.12 \\ \mu_d &= 0.147\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \end{aligned}\right\} for\ M30 \tag{15}$$

$$\left.\begin{aligned} E_d &= 3.94\sqrt{f_{cm}} + 12.02 \\ \mu_d &= 0.142\left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \end{aligned}\right\} for\ M35 \tag{16}$$

Numerous studies [1, 9, 14, 26] investigated the relationships to correlate the Poisson's ratio, moduli of elasticity, density and characteristic compressive strength of concretes. In this study, the following relationships are obtained for elastic moduli and Poisson's ratio of different grades of concretes.

$$\left.\begin{aligned} E_c &= 4.22\rho^{1.5}\sqrt{f_{ck}} \times 10^{-5} \\ E_d &= 5.25\rho^{1.5}\sqrt{f_{ck}} \times 10^{-5} \\ \mu_d &= 6.342\rho^{1.5}\sqrt{f_{ck}} \times 10^{-7} \end{aligned}\right\} for\ M15 \tag{17}$$

$$\left.\begin{aligned} E_c &= 4.16\rho^{1.5}\sqrt{f_{ck}} \times 10^{-5} \\ E_d &= 5.29\rho^{1.5}\sqrt{f_{ck}} \times 10^{-5} \\ \mu_d &= 5.155\rho^{1.5}\sqrt{f_{ck}} \times 10^{-7} \end{aligned}\right\} for\ M20 \tag{18}$$

$$\left.\begin{array}{l} E_c = 4.14\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ E_d = 5.52\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ \mu_d = 4.075\rho^{1.5}\sqrt{f_{ck}}\times 10^{-7} \end{array}\right\} \text{for } M25 \tag{19}$$

$$\left.\begin{array}{l} E_c = 4.12\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ E_d = 5.38\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ \mu_d = 3.537\rho^{1.5}\sqrt{f_{ck}}\times 10^{-7} \end{array}\right\} \text{for } M30 \tag{20}$$

$$\left.\begin{array}{l} E_c = 4.06\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ E_d = 5.25\rho^{1.5}\sqrt{f_{ck}}\times 10^{-5} \\ \mu_d = 3.125\rho^{1.5}\sqrt{f_{ck}}\times 10^{-7} \end{array}\right\} \text{for } M35 \tag{21}$$

Where E_c is in GPa, E_d is in GPa, ρ is in kg/m^3 and f_{ck} is in MPa. The test results can be pooled together from Table 4 in each range of workability for the different grades of concretes to establish the general relationships in between water/cement ratio, compressive strength, Poisson's ratio, and elastic moduli.

The effect of water/cement ratios on the dynamic Poisson's ratio and modulus of elasticity of concretes is studied. Figs. 8 and 9 illustrate the effect of water/cement ratio on the dynamic Poisson's ratio and modulus of elasticity, respectively. It is observed that the modulus of elasticity decreases with the increase in water/cement ratio but the Poisson's ratio is increased with the increase in water/cement ratio. This is because the water has a Poisson's ratio of approximately 0.5. Simple homogenization theory tells that the Poisson's ratio of the mixture lies within the parallel and series bounds of the Poisson's ratio of its constituents. So, the more water mechanically implies the higher Poisson's ratio. Further, Neville [29] reported that the concrete is a composite material comprising of aggregates, cement paste and the interfacial zone between them. Whereas the aggregate-cement interface zone is the weakest link in concrete. If the water/cement ratio is increased, the porosity increases and the aggregate-cement interface zone becomes more porous, and as a result, the higher Poisson's ratio and the lower modulus of elasticity are obtained. In lower water/cement ratio, however, the interface zone becomes stronger. As a result, the lower Poisson's ratio and the higher modulus of elasticity are obtained.

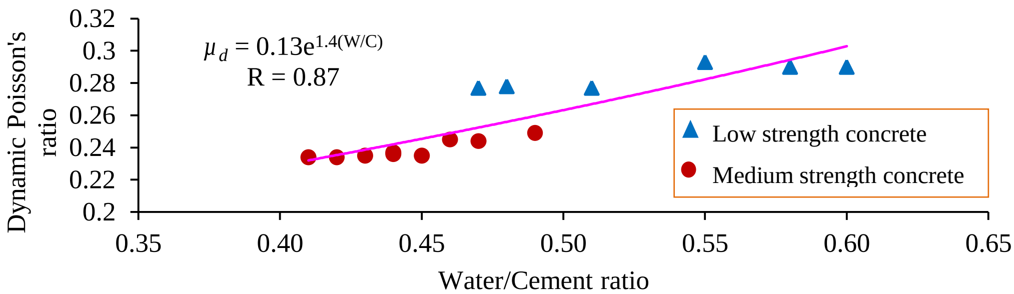

Fig. 8 Effect of water/cement ratio on the dynamic Poisson’s ratio

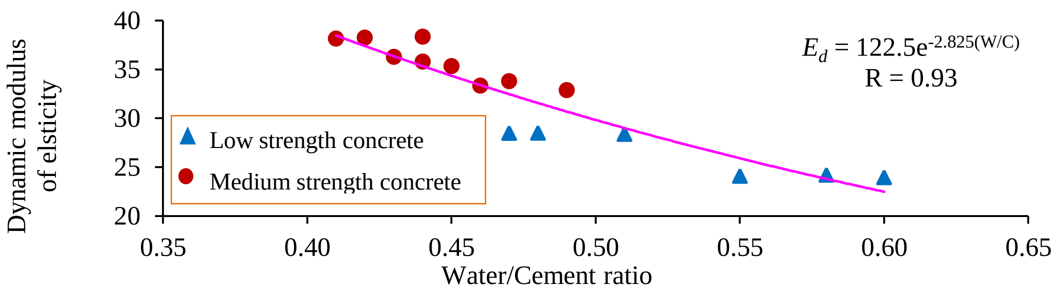

Fig. 9 Effect of water/cement ratio on the dynamic modulus of elasticity

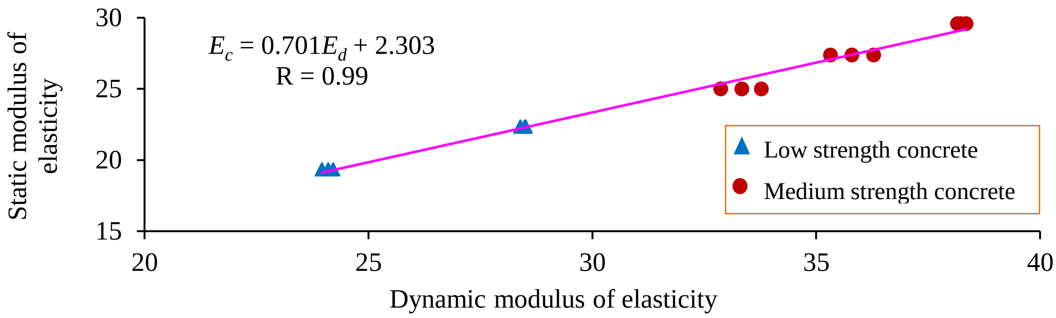

Fig. 10 Relationship between dynamic and static modulus of elasticity

Fig. 10 shows the variation of dynamic and static modulus of elasticity of concretes. The values of static modulus of elasticity are calculated using Eq. (3). It is estimated that the dynamic modulus of elasticity is about 25% and 30% higher than the static modulus of elasticity for medium and low strength concretes, respectively. The similar observations were found in the literature [30], where the dynamic modulus of elasticity was higher than the static modulus of elasticity and was about 20%, 30%, and 40% for high, medium, and low strength concretes, respectively.

The ratio between the static and dynamic modulus of elasticity is higher for a higher strength of concrete. Therefore, the ratio approaches unity when the strength of the concrete increases. In this study, while all the results are accumulated, the simplest relationship between the static and dynamic modulus of elasticity is obtained

$$E_c = 0.78E_d \tag{22}$$

The aforementioned relationship is closed to the results given in Eq. (7). However, the concretes containing less than 500 kg/m^3 of cement, the following expression is obtained

$$E_c = 0.701E_d + 2.303 \tag{23}$$

The relationship between compressive strength and dynamic modulus of elasticity may be established as

$$\left.\begin{aligned} E_d &= 7.035\sqrt{f_{ck}} - 2.82 \\ E_d &= 0.6 f_{cm} + 12.02 \end{aligned}\right\} \tag{24}$$

The relationship between compressive strength and Poisson's ratio may be established as

$$\left.\begin{aligned} \mu_d &= -0.028\sqrt{f_{ck}} + 0.41 \\ \mu_d &= -0.003 f_{cm} + 0.343 \end{aligned}\right\} \tag{25}$$

The relationship between water/cement ratio and dynamic modulus of elasticity is obtained as

$$E_d = 122.5e^{-2.825(\mathrm{W/C})} \tag{26}$$

The relationship between water/cement ratio and dynamic Poisson's ratio is obtained as

$$\mu_d = 0.13e^{1.4(\mathrm{W/C})} \tag{27}$$

The relationship between static and dynamic modulus of elasticity may be expressed in terms of density of the concrete [i.e., Eq. (28)], which is close to the relationship [i.e., Eq. (10)], suggested by Popovics [14]. The aforesaid relationships may be proposed to determine the static and dynamic properties of concretes.

$$E_c = 477.63E_d^{1.4}\rho^{-1} \tag{28}$$

4. Conclusions

Elastic modulus and Poisson's ratio are the important parameters for the structural assessment. This study provides some relevant data on dynamic Poisson's ratio and modulus of elasticity of different grades of concretes with varying water/cement ratio, the density of concrete and compressive strength, which may be useful to the industries. A few relationships are proposed for predicting the values of dynamic Poisson’s ratio and elastic moduli of different grades of concretes. Within the limitations of the test, the following conclusions are drawn from the results obtained.

- The UPV test can effectively be used to determine the dynamic Poisson's ratio and elastic modulus of concretes. A new formulation is proposed for correlating the dynamic modulus and Poisson's ratio. It may be useful to calculate one of the parameters by knowing the other one.
- The static modulus of elasticity can be obtained from the dynamic one. The static properties from the dynamic ones are of interest due to the complexity of performing destructive tests on existing structures. However, the dynamic modulus of elasticity is higher for the higher grade of concrete but the dynamic Poisson's ratio is lower for the higher grade of concrete. The dynamic Poisson's ratio is initially high and decreases with the increase in strength.
- Instead of using a single expression, separate expression for different grades of concretes may give better results in the estimation of dynamic elastic moduli and Poisson's ratio from the compressive strength of the concrete.
- Several existing relationships between the static and dynamic moduli were established. The present study gives a suitable prediction of E_c with an average error of 1.05 GPa while comparing to Eq. (3).
- If the value of mean compressive strength of concrete is known the dynamic values can be obtained from Eqs. (12)-(16), while the Eqs. (17)-(21) can be used for characteristic strength of concrete. These useful mathematical relationships may enable the engineer to predict the Poisson's ratio and elastic moduli of concretes.

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Abbreviations and Symbols

CA	Coarse aggregate
E_c	Static modulus of elasticity, GPa
E_d	Dynamic modulus of elasticity, GPa
FA	Fine aggregate
f_{ck}	Characteristics compressive strength of concrete, MPa
f_{cm}	Mean compressive strength of concrete at 28 days, MPa
NDT	Non-destructive test
ρ	Unit weight of concrete, kg/m^3
PPC	Pozzolana Portland cement
R	Coefficient of determination in regression analysis
μ_d	Dynamic Poisson's ratio
UPV	Ultrasonic pulse velocity
V_p	Longitudinal pulse velocity, km/s
V_s	Shear pulse velocity, km/s
W/C	Water/Cement ratio

Conflicts of Interest

The author declares no conflict of interest.

References

[1] Plain and reinforced concrete-code of practice, Bureau of Indian Standards, IS 456, 2000.
[2] R. N. Swamy, “Dynamic Poisson’s ratio of Portland cement paste, mortar and concrete,” Cement and Concrete Research, vol. 1, no. 5, pp. 559-583, September 1971.
[3] Non-destructive testing of concrete-methods of test, Bureau of Indian Standards, IS 13311 (Part 1), 1992.
[4] J. Carette and S. Staquet, “Monitoring the setting process of mortars by ultrasonic P and S-wave transmission velocity measurement,” Construction and Building Materials, vol. 94, pp. 196-208, October 2015.
[5] Y. Lin, C. P. Lai, and T. Yen, “Prediction of ultrasonic pulse velocity (UPV) in concrete,” ACI Materials Journal, vol. 100, no. 1, pp. 21-28, January-February 2003.
[6] F. D. Lydon and R. V. Balendran, “Some observations on elastic properties of plain concrete,” Cement and Concrete Research, vol. 16, no. 3, pp. 314-324, May 1986.
[7] L. W. Teller, “Elastic properties, in significance of tests and properties of concrete and concrete aggregates,” ASTM Special Technical Publication, No. 169, pp. 94-103, 1956.
[8] J. R. Leslie and W. J. Cheesman, “An ultrasonic method of studying deterioration and cracking in concrete structures,” American Concrete Institute Proceedings, vol. 46, no. 9, pp. 17-36, September 1949.
[9] Building code requirements for structural concrete, ACI 318-14, ACI Standard, September 2014.
[10] M. Kakizaki, H. Edahiro, T. Tochigi, and T. Niki, “Effect of mixing method on mechanical properties and pore structure of ultra high-strength concrete,” ACI Special Publication, vol. 132, pp. 997-1016, May 1992.
[11] T. Takabayashi, “Comparison of dynamic Young’s modulus and static Young’s modulus of concrete,” RILEM International Symposium on Non-destructive Testing of Materials and Structures, vol. 1, January 1954, pp. 34-44.
[12] Design of concrete structures, General rules and rules for buildings, BS EN 1992-1-1:2004, December 2004.
[13] R. N. Swamy and A. K. Bandyopadhyay, “The elastic properties of structural lightweight concrete,” Proceedings of the Institution of Civil Engineers, vol. 59, no. 3, pp. 381-394, September 1975.
[14] S. Popovics, “Verification of relationships between mechanical properties of concrete-like materials,” Materials and Structures, vol. 8, no. 3, pp. 183-191, May 1975.
[15] Xiaobin Lu, Qichen Sun, Wei Feng, and Juntao Tian, “Evaluation of dynamic modulus of elasticity of concrete using impact-echo method,” Construction and Building Materials, vol. 47, pp. 231-239, October 2013.
[16] M. Nematzadeh and M. Naghipour, “Compressive strength and modulus of elasticity of freshly compressed concrete,” Construction and Building Materials, vol. 34, pp. 476-485, September 2012.
[17] Klara Krizova and Rudolf Hela, “Selected technological factors influencing the modulus of elasticity of concrete,” International Journal of Civil and Environmental Engineering, vol. 8, no. 6, pp 593-595, 2014.
[18] Krystian Jurowski and Stefania Grzeszczyk, “The influence of concrete composition on Young's modulus,” Procedia Engineering, vol. 108, pp. 584-591, 2015.
[19] P. S. Kulkarni, A. Ghatge, O. Kank, A. Nair, and R. Aswar, “Experimental investigation on modulus of elasticity of recycled aggregate concrete,” International Journal of Earth Sciences and Engineering, vol. 9, no. 3, pp. 415-419, June 2016.
[20] Byung Jae Lee, Seong-Hoon Kee, Taekeun Oh, and Yun-Yong Kim, “Evaluating the dynamic elastic modulus of concrete using shear-wave velocity measurements,” Advances in Materials Science and Engineering, pp. 1-13, July 2017.
[21] Portland pozzolana cement-specification, Bureau of Indian Standards, IS 1489 (Part 1), 2015.
[22] Coarse and fine aggregate for concrete-specification, Bureau of Indian Standards, IS 383, 2016.
[23] Concrete mix proportioning-guidelines, Bureau of Indian Standards, IS 10262, 2009.
[24] Compression testing machine used for testing of concrete and mortar-requirements, Bureau of Indian Standards, IS 14858, 2000.
[25] R. Jones, Non-destructive testing of concrete, 1st ed. London: Cambridge University Press, 1962.
[26] fib model code 2010, CEB-FIP manual, vol. 2, April 2012.
[27] J. C. Simmons, “Poisson's ratio of concrete: a comparison of dynamic and static measurements,” Magazine of Concrete Research, vol. 7, no. 21, pp. 61-68, July 1955.
[28] H. E. Krenchel, “Structure concrete and its behavior under Load,” Proc. International Conference, September 1965, pp. 160.

[29] A. M. Neville, Properties of concrete, 3rd ed. England: Longman Scientific and Technical, Essex, 2002.
[30] P. K. Mehta and P. J. M. Monteiro, Concrete microstructure, properties, and materials, 3rd ed. New York: McGraw-Hill, 2006.