International Journal of Engineering and Technology Innovation, vol. 13, no. 3, 2023, pp. 230-250

Shoe Last Customization: A Systematic Review

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Received 31 December 2022; received in revised form 24 March 2023; accepted 28 March 2023

DOI: https://doi.org/10.46604/ijeti.2023.11348

Abstract

In recent years, there is an increase in research into shoe last customization and topic analysis methods. The work aims to systematically review the literature on the customization of shoe lasts. The method used in this work is to perform a five-phase systematic review algorithm. Data on the research performed are extracted and synthesized from each study: main research objectives, authors, date of publication, journal, or conference in which the article was published, and the quality of each article. The studies included in the review are published between 2018 and 2022. The results of the review are nineteen papers about the process of customization of the shoe last. The conclusions of the analysis indicate that the quality of research has not changed over time, in 2020 there was a decrease in work. Most often, researchers analyze the impact of anthropometric factors on the correct shoe last modeling and methods of shoe last parameterization.

Keywords: shoe last, customization, 3D design process, CAD technology, reverse engineering

1. Introduction

The development of shoe design and shoe lasts have a very rich history. In the article [1], Pelizzari included a description of the various stages in the development of shoe production and design: from sketches of the individual shoe cuts through the selection of materials for the various parts of the shoe. The development of shoemaking was also concerned with the design of the various components of the shoe: the sole, insoles, heels, cushioning, and other factors affecting comfort and use. All these parts form a structure that plays a very important role in our lives - but customers don't realize how important this structure is. There are a lot of factors that depend on the right choice of footwear (both in terms of comfort and dimensions). Wrong footwear size or shape mismatch can cause irreversible changes in human bone structures and lead to musculoskeletal dysfunctions and postural defects associated with pain [2].

In the research conducted by Buldt and Menz [3], you can see how a significant number of the population uses the wrong size of shoes. Participants in the studies were of different nationalities and the research was conducted independently. It does not include only developing countries.

Because this is a problem so common that the authors analyzed the factors affecting the wrong selection of footwear. The study found that 63-72% of participants wore shoes that did not fit their dimensions. On the other hand, older people and people with diabetes were more likely to wear shoes that were too narrow (between 46 and 81%). [3]. Foot pressure during gait while using footwear is also an important factor - Tatjana proposes a method to achieve sole personalization according to the plantar pressure maps for each foot [4], which leads to even foot pressure, which to some extent can eliminate pain and discomfort during prolonged walking or standing.

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English language proofreader: Chih-Wen Teng

The footwear manufacturing process consists of more than a dozen steps and is very complex. It is possible to distinguish 14 sub-processes during the creation of new footwear [5]:

(0) Footwear design concept - design of appearance and cut.

(1) Selection of material for the upper

(2) Leather Cutting and edge shaving

(3) Edge folding

(4) Stitching on different pieces

(5) Upper stitching finished

(6) Lace Hole

(7) Insole nailing

(8) Toe box and counter applying

(9) Fitting and upper on the last

(10) Toe lasting

(11) Instep lasting

(12) Heel lasting and fixing

(13) Shank applying

(14) Fixing of shoe

The zero phase concerns the creation of the idea - the design of the shoe - visually - most often it is the designer's sketch, which includes the final effect of the appearance of the shoe. Based on the selected style, cut, and type of footwear, the next stages of manufacturing can begin. The first stage is the selection of the material from which the footwear will be made. The next stages are the cutting of individual elements (most often it is cutting elements from a sheet of leather or synthetic materials) - these are stages 2 and 3. The next stages are about stitching the individual elements together - these are stages number 4,5,6. The individual parts are joined together and form the elements that will be applied to the shoe last.

From stage 6 onward, the basic element of the manufacturing process is the shoe last - it is its shape and dimensions that determine how the shoe will be formed. It is through it that the designer can merge all the other elements into a single upper and connect it to the sole. As you can see, without the shoe last it would be impossible to produce a shoe. As demonstrated, the shoe last is an essential element in the manufacturing process. This proves that its modeling and design are very important and play a key role in ensuring the proper shape and dimensions of shoes.

In the shoemaking process, the most important part is the shoe last. It is the starting point of every shoe design. This is for a simple reason: correctly constructed and sized lasts will ensure optimal shoe design. The shoemaker starts work with the last and creates the shoe outward. Traditionally, the designer creates a new shoe model and prepares technical documentation for the shoe last mechanical production. Combining the artistic skills involved in modeling the shape of the shoe last and the technical knowledge involved in correct construction – shoe last designers have utility skills. The rapid growth of 3D modeling and scanning enables support for research related to the design of lasts and their adaptation to footwear users. The existing methods of shoe last construction have achieved new possibilities. They are related to both the design and production of footwear. This sector is interested in manufacturing implementation.

2. Systematic Literature Review

Systematic literature review The systematic review is an approach for synthesizing research by following a strict, pre-defined algorithm aimed at reducing bias. Currently, the amount of scientific literature is so vast and easily accessible that it is tough to read all the studies separately. This makes it difficult to find clear answers to research questions [6-7]. The situation is simpler when there is access to systematic literature reviews, i.e., they focus on a formulated research question and use precisely described, scientific methods of identifying, evaluating, and synthesizing all sources relevant to the research problem. The work on a systematic review can be divided into five-phase (Fig. 1).

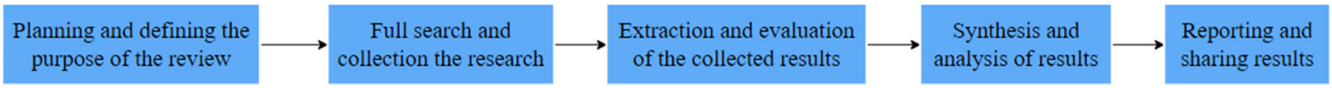

Fig. 1 A five-phase systematic review procedure [1]

The planning activity prepares researchers for the review, which forces them to think about the aims of the project, its stages, and related tasks. The purpose of the initial literature search is to check for gaps in systematic research and to get an idea of the scope of literature on the topic of interest and what scientific evidence is available [6]. Another task within the initial search is to determine the criteria according to which the selection of literature in the full search will be made. The inclusion and exclusion criteria refer to the characteristics of the studies, for example, study participants, type of study, how the measurements were taken, and how they were validated. In preparation for a full search, extract keywords based on the known, most relevant to the research question of the article. Extract terms from the article (e.g., keywords from the title of the article). Check what terms the article is indexed within the database. The entire literature search process has five-phases [7]:

(1) Framing questions for a review

(2) Identifying relevant work

(3) Assessing the quality of studies

(4) Summarizing the evidence

(5) Interpreting the findings

Nowadays, formalizing a single approach to the shoe last customization is an impossible assignment to achieve. The adopted method must consider different parameters as the way of foot measurement, a tool to do that, using existing databases or creating your parametric system, etc. In this context, it is essential not only to focus on the system’s functionalities, accounting for its dependencies but also to consider all those quality aspects that may be impacted by the model of shoe last.

3. Shoe Last – 3D Modeling and Product Customization

Even if the COVID-19 pandemic still had an impact on some nations' industries, more than 22 billion pairs of shoes were produced globally in 2021, an 8.6% increase. Production is still 2 billion pairs below the pre-pandemic level, though. The pandemic did not halt the trend toward spatial consolidation of the footwear industry at a wide, continental level. Today, Asia produces more than 88% of the world's output, up more than 0.5 percentage points from the previous year. Moving oppositely, Africa's share, and Europe's both fell below 3%. Most of the manufacturing outside of Asia is in South America, which saw a minor increase in its proportion [8].

Major shoe manufacturers with vast experience in shoe design must have expertise in 3D modeling and mass manufacturing. Knowledge of the theory and construction basics is not enough, marketing knowledge is also required, which will integrate the functionality obtained with the help of designers and customization as well as the appearance and design resulting from the marketing features of the product and the sale of goods [9]. The 3D design has greatly accelerated production processes: shoe design and large-scale shoe manufacturing. Integrated CAD/CAM software is gaining a growing following among global shoe manufacturers due to its vast 3D design capabilities. The CAD/CAM program provides footwear manufacturers with end-to-end solutions: from design to manufacturing. CAD modeling enables "solid-surface hybrid modeling" design and allows users to use all functions on both solid and flat surfaces.

The option to combine modeling on solids and surfaces makes the design process more flexible and improves efficiency, whether designing a sole, a shoe last, or an entire shoe. Modeling the shoe last is a basic task when designing shoes. With CAD software, such elements can be imported and stored in a library. The use of 3D scanning and reverse engineering is also an important element in the process of creating shoe lasts and shoes [3].

Today's technology supports the processes of scanning and data acquisition, using, among others, smart data, and transferring data in the cloud. Rafiq et al. [10] have developed and implemented a smartphone application based on computer vision software that provides the functionality to automatically measure the four basic dimensions (length, width, arch height, and instep circumference) of the human foot from 3D images and scans. As research shows, the accuracy obtained from the measurements oscillates between 95 and 99%. Scientists consider this a satisfactory result. Analysis of the availability of applications for measuring foot parameters using mobile devices [11] showed that 26 applications make this possible.

However, a significant number of applications did not meet the requirements set by the researchers. They found that 6 of the 26 apps they analyzed reportedly used external scanners or advanced algorithms to reconstruct 3D models of the user's foot. The implication is that currently, available foot measurement apps do not follow any specific measurement guidelines.

3D scanning is a process for which either an optical or laser 3D scanner is used, and the result is a triangle mesh. The triangle mesh virtually represents the actual geometry along with color (i.e., texture). Depending on the 3D scanner used, triangle meshes with different resolutions can be obtained – it all depends on needs. Transfer of a product of this kind - with complex structure, unconventional dimensions, and complicated colors, to the virtual world, is much cheaper and faster with the help of a 3D scanning service, than with manual measurements and cooperation with a computer graphic designer [12].

Another application is the reverse engineering of footwear. It is useful when 3D documentation of the shoe last being created is needed - first, a 3D measurement of the customer's foot should be taken, so that a CAD model can then be made based on it, or, if sufficient, the corresponding flat sections. In addition to 3D scanning of the hoof itself, it is also possible to make a 3D scan of the shoe last and, based on such a result, make a CAD model of the shoe by using reverse engineering [3].

3.1. Customization of product

Nowadays, to succeed in the market, a company must focus on customer relationships and individual needs. The key here turns out to be the establishment of a special, unique relationship with each customer. In marketing, this is called a one-to-one marketing relationship. In this case, it is not important to mass market to many buyers, but to establish an individual relationship with each of them and present them with a unique, tailored offer.

Product customization is a strategy based on the production of individualized products that are eligible for the specificity of customers at a reasonable price. The products are user-oriented, and the key element is co-designing the product with the client - as part of the selection of options or predefined components (in the case of footwear production) or anthropometric measurements (for shoe last). This sector is interested in implementing 3D scanning tools, 3D printing, robotization, machine learning, and artificial intelligence into production. They allow for improving the construction process and ensure better adjustment of dimensions to the shape of the shoe last.

The concept of product customization is an increasingly used method to ensure uniqueness and fulfill the customer's requirements. An important aim of customization in the Industry 4.0 concept is to fit the product to functional and ergonomic requirements. The concept of mass personalization has become one of the issues in the theory and practice of production management. Customers demand an increased variety of products while maintaining a favorable selling price. Therefore, manufacturers must develop technologies and strategies to provide goods and services that meet individual customer needs at low or no production costs. This is only possible when production processes are organized with an efficiency approaching mass production. The difficulties associated with such a process are described by Boër and Dulio [13].

With anthropometric measurements, product personalization reaches a much higher level of technological sophistication. Using computer software, you can design a model that, after computer simulation, can be continuously optimized and does not require a physical prototype. Increasingly, customization is also being used in footwear. A significant quantity of automatization has been achieved by the implementation of the CAD system in footwear production [3].

Consumer demand for the shoes they buy is increasing. However, there is little awareness related to choosing the right footwear and the consequences of using ill-fitting footwear. Research by Teyhen et al. [14] shows that 35% of participants have incorrectly sized shoes, and 35% use extremely worn shoes, which could contribute to injury as well. According to Shyam and Singh [15], customer awareness of the environmental impact of footwear materials is also low. Depending on the parameters studied, it reaches results of around 30-40% of the group surveyed.

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3.2. Shoe last modeling research review

The design and production of shoe lasts has been improved in several ways, according to published literature. Foot morphology has drawn more and more attention from biomechanics researchers and shoe manufacturers. Generally, 2D footprints are used to characterize the morphology of the foot. Furthermore, because the vertical dimension of the foot is disregarded in footprint quantification, it is impossible to accurately measure the complicated 3D geometry of the foot.

By examining and categorizing human body types, Mochimaru et al. [16] suggested that it would be possible to build things that fit people properly. Deforming the shapes of the foot by shifting control lattice points placed around the item and recoding the movements as dissimilarity was done using the Free Form Deformation (FFD) technique to study human body forms. The design of the shoe lasts was one example given. Another solution was a grouping approach for 2D/3D products was put forward by Kim et al. [17]. By encoding the convex hull into a list of 2-tuples based on the volumes, the convex hull of a chosen base product was rotated virtually to determine its shape and general similarity to the other products. The values of similarity were then used for grouping. To categorize similar shoe lasts, a prototype system had been employed. Under very tight assumptions, this procedure is workable for the custom-tailored shoe lasts. It cannot categorize shoe lasts of various styles, nor does it speed up or simplify the process of designing a custom shoe last.

Customization significantly affects the design, production, and sales processes. Manufacturing must accommodate an endless range of dimensions; thus, design workflows must be highly optimized to produce specific products for each customer. A technique to customize a shoe last for a user was created by Japanese researchers Takahashi et al. [18]. They measure the feet and digitize the shoe last. The foundation shoe last is then adjusted manually using Laplacian deformation to conform to the design limitations and fit the foot's measurements.

Despite the potential information loss along the vertical axis, analyzing the two-dimensional footprint is a standard method for studying foot morphology. The fact that footprints may be obtained, measured, and maintained with the use of wax, plaster, foam, or dynamic pressure plates accounts for their widespread use.

Foot morphology has got increasing interest from biomechanics researchers and shoe manufacturers. Commonly, 2D footprints are used to characterize the morphology of the foot. Furthermore, because the vertical dimension of the foot is disregarded in footprint quantification, it is impossible to accurately measure the complicated 3D geometry of the foot.

4. The Human Foot and the Shoe Last - Main Differences

The shape and construction of the last determine the aesthetic shape of the footwear and its functionality. The last should consider the details of the structure of the foot and, changes during gait. The modern shoe last is not a copy of the human foot. Making the last must be preceded by foot measurements. The measurement results are used in the last construction process. For the designer, this is the most important part of the project. Then, the foot measurements are converted to shoe last parameters. Even though the last of a shoe is a complex 3D shape, manufacturers use several reference points for their design [19-20]: vamp, instep, ball break, and heel point.

In addition, basic measurements and circumference are used:

(1) Last length – measured from the heel seat to the toe point.
(2) Ball girth – measured around the last at the widest point of the fore part.
(3) Waist girth – measured from the midpoint between the instep girth and ball girth lines to the middle of the cone
(4) Instep girth – measured from the shank to the top of the last’s cone
(5) Long heel girth – measured from the last’s heel counterpoint, across the cuboid, to the vamp point on the main axis
(6) Short heel girth – measured from the last’s heel feather edge, across the cuboid, to the point where the instep

Although the shoe last is a representation of the human foot, some differences distinguish them from each other. The comparative analysis [22-25] is presented in Table 1.

Table 1 Comparison of the basic characteristics of the shoe's last and the human foot

Feature	Shoe last	Foot
Purpose	Shoemaker's basic tool, Used to produce shoes - as a model of the human foot	Weight-bearing Adapt to the ground, load, and movements
Shape	Regular and continuous outline of the shoe last sharp feather edge	Irregular Without sharp edges
Feather	Increase gradually height from the feather line	Not present
Surfaces	Smooth to enhance the appearance and shape of the shoe	Irregular, vary with individuals
Texture	Hard, strong	Soft, flexible
Heel height	Curvature for modeling heels	Does not exist
The front part	Thinner to help the shoe to grip the foot to prevent pressure on the foot	Depends on the anatomical structure of the foot
Length	Greater than a foot to prevent pressure on the foot	
Toe spring	Improves the modeling of the shoe.	Not present
Girth and size	Regular, the same for left and right	The right and left foot usually have different dimensions depending on the anatomy

5. The Role of Customization in Shoe Last Design

Traditional shoe production for the customer is difficult and time-consuming. It depends heavily on the skill and experience of the designer. Footwear manufacturers must manually measure the shape of the customer's foot (usually about thirty measurements). Then the designer must make the wooden model by hand, during which time experience is especially important. Moreover, trace and error modifications are often necessary.

Recently, with the development of 3D scan technology, it is possible to automate the process to make custom orders, as information about the 3D foot shape is easily retrieved. A simple idea is to replace some of the latest shoe models with a model matching the scanned foot data.

Custom-fit means personalized in terms of shape and size. Personalized products include modification of some of their features according to customer requirements. The concept of customization can be understood as a form of offering one-of-a-kind products that can be completely customized to geometric features due to their basic characteristics and application. Customization is the involvement of end users in the way the product is developed and manufactured. There are three types of customizations [26]:

(1) Soft - The customer chooses from existing basic products.
(2) Hard - The product parts are selected from prefabricated parts by the customer.
(3) Made to order: All components of the product are manufactured upon consumers' specifications and requirements.

For each type of personalization, the following shoe values can be optimized for the end customer:

(1) A shoe that is designed to satisfy the performance requirements of customers.
(2) The customer configures footwear fashion by selecting fabrics, colors, or accessories for the basic shoe design.
(3) This type involves producing a shoe that the end user will wear without pain and discomfort, usually requiring special last and additional measurements.

There are two main approaches to making a customized shoe last design. The first is about automatically modeling a customized shoe last based on scanned foot data and/or manual measurements and comparing it to a scanned shoe last data, giving the design like the last and fitting with the foot shape. The second method is to create a model of the shoe last based only on the scanned foot and use a parametric date and algorithm to create a new custom shoe last. A detailed analysis of the discussed solutions will be presented later in the article.

6. Research Method

The research described in the review is a global shoe last customization development. In this context, it is important not only to focus on the customization method's functionalities, accounting for its dependencies and responsibilities but also to consider all those quality factors that may be impacted by the shoe last design.

The purpose of conducting the review is to systematize knowledge of the methods of the design process and customization of shoe lasts. The author of this article's interests in this topic are closely related to design and modeling with consideration of reverse engineering elements and 3D scanning, and has mainly focused on the design of atomized shoe insoles [27-28].

Due to the growing interest in the topics related to the doctoral thesis related to the automation of the design and construction process of the shoe last, the author decided to conduct a systematic review. The development of technology and CAD software necessitates the improvement of existing solutions and the discovery of new ones, which thanks to technology are possible to implement (and only a few years ago required the preparation of algorithms or manual measurements - some of the improvements are already implemented into the software).

Therefore, it was decided to review methods from the last 5 years only - for the sake of discerning whether more modern methods have made it possible to mass-produce shoe lasts for individual customers in a faster, cheaper, and more widespread way.

6.1. Research questions

The systematic review aims to determine the efficiency of the designing process of shoes last and its customization. The aims are to:

(1) Systematically search the published literature to identify studies that have explored the method of constructing the shoe last customized for customers.

(2) Explore the development of methods and quality of measurements and last customization of the available research literature concerning children's therapeutic footwear.
(3) Determine the benefits or negative effects of the shoe last customization process.

Based on the aims of this work, the research questions were selected. Table 2 also includes a description related to the motivations for focusing on these topics.

Table 2 Research questions (RQ) with explanation

ID	Question	Motivation
RQ1	Where has the research on the shoe last customization been published?	Identify the worldwide trend of research sites.
RQ2	What is the annual number of shoe last customization studies?	Analyze the number of studies each year related to the designing of custom shoes last
RQ3	Which types of feet were most often chosen for the shoe last modeling?	Definition of the study population and scope of the research
RQ4	Which research implements the last modeling based on 3D measurements of customers?	Determine methods using 3D modeling and approaches more modern than manual design.
RQ5	Which scientific studies consider factors related to changes in foot dimensions resulting from loading, fatigue, and external factors?	Detect methods for detecting the impact of external factors on the change in foot size.
RQ6	What are the strengths, limitations, and weaknesses of the study?	Finding opportunities for further research and improvement of the customization process.

Questions RQ1 and RQ2 involve general information - statistical and geographic about published works. Questions RQ3-5 are concerned with methodology as an aspect most important to the study. RQ6 is related to the strengths and weaknesses of the methods and studies that were analyzed.

6.2. Inclusion/exclusion criteria

The selection procedure of the found works was performed using inclusion and exclusion criteria. Inclusion criteria:

(1) Papers published from 2018 to 2022.
(2) Paper in English.
(3) PDF Available.
(4) Were published in, or submitted to, a conference or journal, or were technical reports or book chapters.
(5) The journal article describes how to collect data, the methods used, and the contributions made to the literature.

Exclusion Criteria:

(1) Informal literature surveys (no defined search questions, no search process, no defined data extraction, or data analysis process).
(2) Not related study.
(3) Patent.
(4) Footwear\sole customization.
(5) The selected works were then further analyzed.

6.3. Data sources and search strategy

The literature search method included 2 main aspects related to the interests of the author of this paper in conducting research. The first was to find research that solved the problem of automating the process of shoe last construction. As described above, the human foot is a very complicated element to map and transform as a shoe last. One of the problems that occur during 3D scanning is the complete freedom of positioning the triangle mesh model with respect to the axis of the coordinate system - the model of the scanned foot is imported into the CAD program in any orientation. Therefore, it was also sought to find works that somehow solve this problem and automate it. Another aspect was to search for solutions to improve the automation of the customization process - to see if there is an algorithm that can massively match the geometry of the foot to the model of the designed shoe last.

As a result, the number of studies included in the review is small, because the subject matter is very narrow and not common, such as the mass production of shoes. However, the topic is relevant, as written in the first part of this article. During a search of Science Direct databases, as many as 375 items were found, but their content largely lacked information on shoe lasts and their modeling process, so they were not included in the review. The quality of filtering records based on keywords; this database proved to be the least effective.

The research method applied in this survey consists of five-phase described in the earlier part of this work (Fig. 1). At the planning stage, after defining the research questions, the search key was compiled based on the identified keywords:

"Shoe last" OR "last design" OR "foot last fitting" OR "Shoe last customization" OR "customization of shoe last" OR "shoe lasts" OR "design shoe lasts" OR "design shoe last"

A systematic search was conducted, resulting in 581 papers. The most relevant manuscripts were selected based on the established subtitles of the review (modeling methods, and customization methods considering both structural features of footwear and physiological and ergonomic factors). Four databases were used to search for records: Scopus, Taylor, and Francis, Science Direct, and Web of Science. After the removal of duplicates, documents were screened by title and abstract. There were sixty-seven relevant documents, and after the full-text screening, fifteen documents were included. By using the snowballing approach, four more papers were identified, which provided a total of nineteen articles used in this review. Fig. 2 shows a flow chart of searching and its phases.

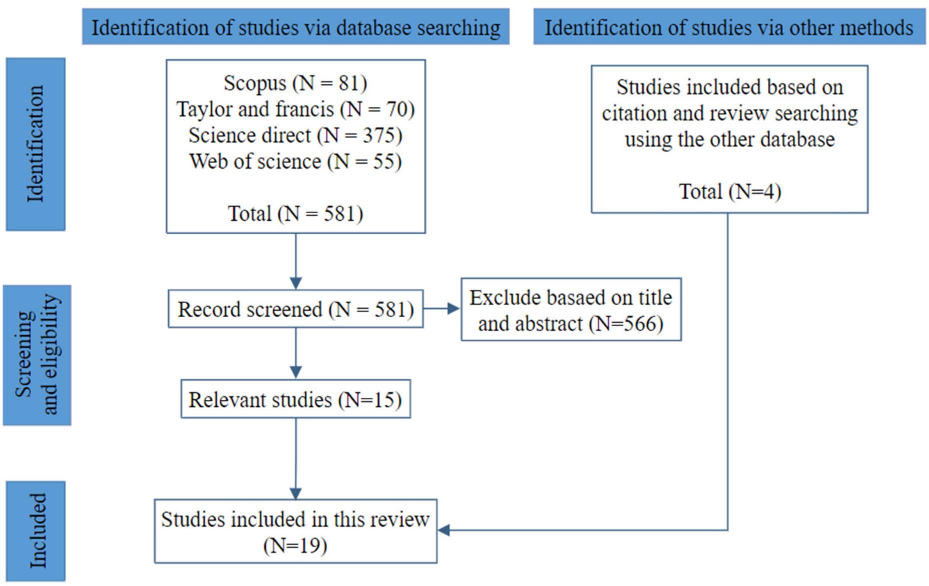

Fig. 2 The review flow chart

6.4. Quality assessment

(1) (A) - Is the aim of the study clearly described?

(2) (B) - Are the main results to be measured clearly described in the introduction or methods section?

(3) (C) - Are the participants' characteristics in the study clearly described?

(4) (D) - Are the measurements valid, accurate, and statistically significant?

(5) (E) - Does the study explain the algorithm used to model the shoe last?

(6) (F) - Does the article clearly describe the last customization process?

(7) (G) - Are the conclusions based on accurate interpretations of the data?

The criteria are based on seven questions and were scored as follows:

(1) **Criterion A:**

score 1 – the purpose of the work is clearly defined in the work.

score 0.5 – the goal is concludable.

score 0 – goals are not defined and cannot be easily deduced.

(2) **Criterion B:**

score 1 – The results are clearly defined, and the measurement process is discussed.

score 0.5 – Results are generalized without specifying the measurement process.

score 0 – The results are not defined and cannot be easily inferred.

(3) **Criterion C:**

score 1 – Participants are described in detail - both quantitatively and qualitatively.

score 0.5 – Participants in the study are defined, without their characteristics.

score 0 – Participants are not defined and described.

(4) **Criterion D:**

score 1 - Measurements were conducted accurately - their description and research method were included.

score 0.5 - Measurements were made, but their description was not sufficient.

score 0 - Measurements not taken.

(5) **Criterion E:**

score 1 - The customization algorithm is characterized and described in detail.

score 0.5 - The customization algorithm is present but not described.

score 0 - Algorithm not present.

(6) **Criterion F:**

score 1 - The customization process is characterized and described in detail.

score 0.5 - The customization process is described in general terms.

score 0 - Process description not present.

(7) **Criterion G:**

score 1 - Conclusions are detailed and accurately characterized based on the obtained test results.

score 0.5 - Conclusions exist, but they are general.

score 0 - Conclusions do not exist or do not involve the completed research.

Table 3 summarizes the results of the assessment of the Quality Criteria.

Table 3 Qualitative analysis results

No.	Quality assessment-criteria							Total
	A	B	C	D	E	F	G	
A01 [24]	1	1	1	1	1	1	1	7
A02 [25]	1	1	0	1	0,5	0,5	1	5
A03 [26]	1	1	1	0,5	0,5	0	1	5

Table 3 Qualitative analysis results (continued)

No.	Quality assessment-criteria							Total
	A	B	C	D	E	F	G	
A04 [27]	1	0,5	0	1	0,5	0,5	0,5	4
A05 [28]	1	1	0	1	1	1	0,5	5,5
A06 [29]	1	1	1	1	1	1	1	7
A07 [30]	1	1	0	1	1	1	1	6
A08 [31]	1	1	1	1	1	1	1	7
A09 [32]	1	1	0,5	1	1	1	1	6,5
A10 [34]	1	1	0	1	1	1	1	6
A11 [35]	1	1	0	1	1	1	1	6
A12 [36]	1	1	0,5	0,5	0,5	0,5	1	5
A13 [37]	1	1	0	1	1	1	1	6
A14 [38]	1	1	0	1	0,5	0,5	1	5
A15 [39]	0,5	1	1	1	1	1	1	6,5
A16 [40]	1	1	1	1	1	1	1	7
A17 [41]	1	0,5	0	1	1	1	0,5	5
A18 [42]	1	1	0	1	1	1	1	6
A19 [43]	1	1	0,5	1	1	1	1	6,5

7. Results

The data synthesis aims to summarize the data obtained from the primary research to analyze the current trend of the shoe last customization process. This section reports the results of each research question.

7.1. RQ1: Where has the research on the shoe last customization been published?

Table 4 Distribution of research papers by country.

Country	Number of studies	%
Romania	3	16%
Netherlands	1	5%
Ukraine	4	21%
Indonesia	2	11%
Mexico	1	5%
Taiwan	1	5%
Australia	1	5%
India	1	5%
France	1	5%
Germany	1	5%
China	2	11%
Vietnam	1	5%

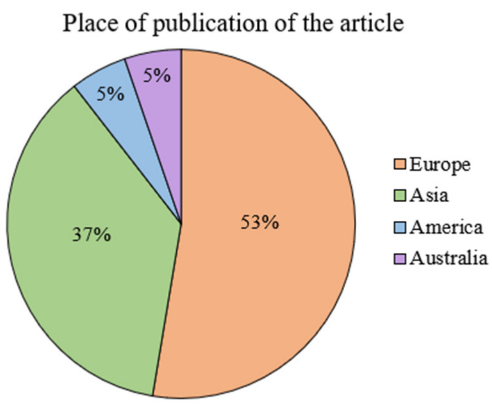

Fig. 3 Distribution of research work by continent

Table 4 shows the number of countries where work on footwear customization has been published. However, the issues among them are quite diverse, with research being conducted in different fields and using different solutions to the research problem. The decomposition of publications by continent is also presented. In this case, the largest portion of the research is conducted in Europe and Asia (Fig. 3).

7.2. RQ2: What is the annual number of shoe last customization studies?

The next question invited the distribution of the number of papers relative to the years in which the systematic literature review was conducted. The years from 2018 to 2022 were included (Fig. 4). The highest number of papers was published in 2019, followed by a decline in publications. The consequence of this result may be the ongoing COVID-19 pandemic. In 2022, the number of publications increased again, generating renewed interest in the topic among scientists around the world. Fig. 5 provides information on the distribution of research conducted by country. In 2019, a very large increase in the number of publications and national diversity can be observed, which may indicate that many independent studies on the topic of shoe last formation are being conducted simultaneously.

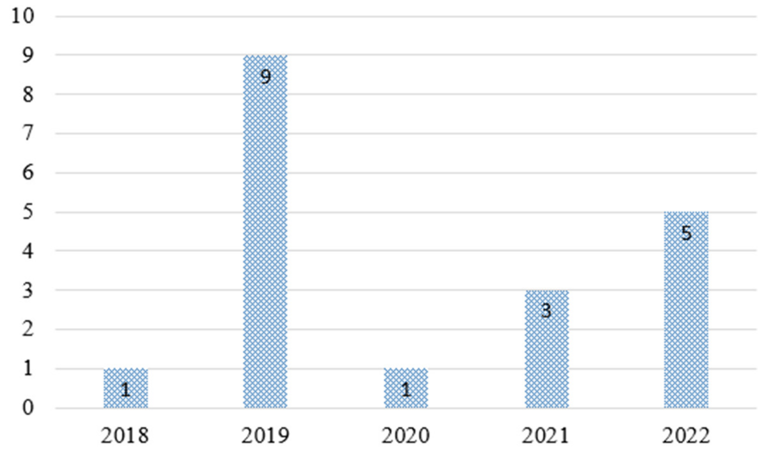

Fig. 4 Distribution of research work by years

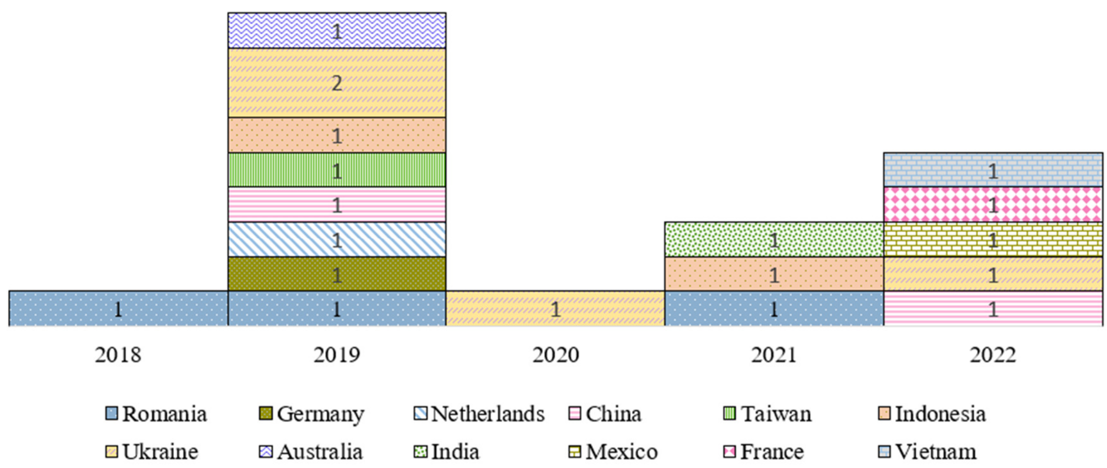

Fig. 5 Country distribution of research work by years

The following years marked the beginning of a crisis in publications, however, the upward trend resulted in the fact that in 2022 there was a significant view from publications compared to previous years. In Fig. 6, shows a division by continents, where in 2019 Europe had a significant impact on the research of the shoe last modeling. Then, in the following years, this trend continues, however, the number of publications in Europe has decreased significantly. It is worth noting that countries in America and Australia have also conducted relevant research on this topic. This means that the topic is very intensively developed both in European and Asian areas as well as in other developing countries. Asia maintains a tendency of a stable level to publications and conducted research, which may indicate a continuous interest in the subject.

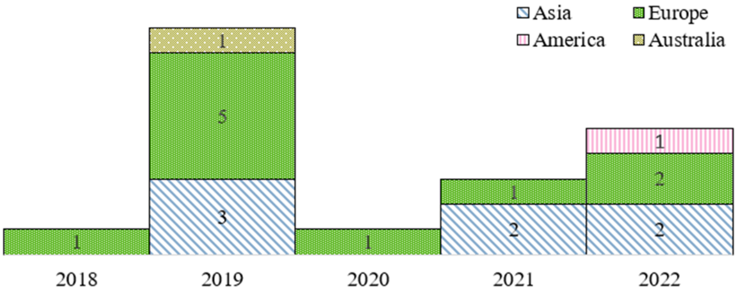

Fig. 6 Continental distribution of research work by years

7.3. *RQ3*: *Which types of feet were most often chosen for the shoe last modeling*?

Research from the shoe last customization process takes place in two fields. The first concerns the modeling of the shoe last for healthy feet, and the second considers the dysfunctions that occur with the feet, caused by poor sizing or congenital foot diseases, among other factors. Shoe last for healthy feet is modeled in the works [30-34, 36-42]. The study by Pang et al. [37] included adaptation to the athlete's foot. The project included a shoe last adapted to cycling shoes.

The paper discusses the main aspects of the requirements for this type of footwear and proposes a shoe last modeling algorithm. Other works focus on foot dysfunction. Most of them deal with diabetes [37, 46]. All the articles highlight the problem of ill-fitting footwear for this type of dysfunction. However, the solutions proposed in them significantly improve the comfort of use and even slightly improve the visual appearance of this type of footwear. The work of Villarreal-Calva et al. [35] developed a footwear model for clubfoot. Clubfoot is a birth defect in which one or both feet are rotated inward and downward [48]. The process of adapting the shoe and shoe last to such a foot is extremely difficult due to the aspect of comfort, ergonomics, and biomechanics, but also visually. Using Curve Base Surface Modeling, a model of the shoe last was satisfactorily obtained and a prototype of the shoe was made.

7.4. *RQ4*: *Which research implements the last modeling based on 3D measurements of customers*?

This research question aims to acquire knowledge that includes an overview of data acquisition methods for creating a shoe last model. As a rule, this is anthropometric data including basic measurements of the client's feet - lengths, girths, and, in some cases, consideration of biomechanical, ergonomic, and aesthetic parameters [34-36, 38-40]. Data can be acquired manually using a ruler or a Brannock instrument when capturing customer foot measurements. Increasingly, data is being acquired using computer-aided design techniques. 3D scanning and CAD surface modeling are being used to measure feet [30, 35-40, 42-43, 45]. This form is easier and less time-consuming than traditional measurements.

However, to use this type of tool, it is necessary to have adequate knowledge of how to operate the scanner and 3D modeling. In addition, these devices are much more expensive compared to traditional measuring tools used by a shoemaker. Another aspect that was analyzed in this part of the work was methods related to the way anthropometric data is processed into a shoe last model. In general, one can observe a tendency to use several methods in this aspect. The first concerns the modeling of the customized shoe last based on the last measurement data obtained from databases. This is the so-called fitting of the shoe last to the reference of the base shoe last - usually based on predetermined reference points [37-39, 45]. The second method involves creating algorithms or using machine learning, neural network, or artificial intelligence [33, 36] to create shoe lasts based on curvatures that match the human foot as closely as possible. The last method is surface modeling based on the foot model from scan [30, 35, 40, 42-43].

7.4.1. *Methods based on shoe last re-design*

The use of anthropometric data and the re-design method is often used by researchers. Re-design involves modeling footwear regarding base models but considering the basic parameters of the feet of the customers for whom they are made.

Amza et al. [30] use an algorithm to model a new shoe last by acquiring anthropometric data (by scanning) and a re-design of the shoe last, which was obtained from databases. Reference points and a foot positioning algorithm relative to the coordinate system were used for proper modeling and positioning. The technique used the Free Form Deformation techniques. Methods for positioning the model after scanning relative to the coordinate system were used by Wang et al. [36]. Spline curves and surface modeling were used in the shape-fitting technique. The study also includes the use of reference points to improve the footwear modeling process.

7.4.2. Methods based on parametric modeling

Parameterization of the model is helpful in the case of process automation or mass customization of products. In the case of footwear, this process is exceedingly difficult due to the lack of standardization in the process of obtaining dimensions and difficulties in modeling the shoe last based on the shape of the foot as a non-geometric model. The following section of the article investigates the possibility of parameterizing the shoe last model.

In the study conducted by Booth et al. [31], one of the issues is the 3D printing technology and its aspect related to the design of the shoe last to properly adapt its model to the customer's foot. For this purpose, a series of measurements of the customer's foot was used. To properly adapt the shoe last model to the customer's foot, it was used to take several measurements of the customer's foot and input into the parametric model. This method is accomplished by replacing the intersection curves with parameterized spline curves.

Another author, Tian et al. [44], approaches model parameterization issues. Their method follows the molding characteristics of the longitudinal section of the shoe last and combines with the Grasshopper parametric design plug-in to construct an automatic last profile forming program.

In the study conducted by Zhang et al. [45], parametric pattern-making technology based on AutoCAD parametric drawing technology was assumed to achieve mass customization of the shoe bottom pattern. Based on the analysis of the geometric features of the shoe bottom pattern, the pattern architecture and dimensions were parameterized using the geometric tools and dimensional constraint tools in AutoCAD, resulting in a parametric shoe bottom pattern model.

7.4.3. Methods using surface modeling and spline curves

Most often, surface modeling and the use of spline curves are used to model shoe last. As a result, customer data can be used to optimally adapt the model.

The study by Villarreal-Calva et al. [35] used a similar method - using a Curve Base Surface. In addition, an algorithm was developed to transform the foot dimension into the final shoe model. However, this method does not take into consideration the parameterization of the model due to the specifics of the foot with dysfunction.

Sinha [38] has applied automatic extraction of foot features. This study identified eighteen key foot parameters (five lengths, four widths, three heights, and six circumferences) which were adjusted to the design requirements for a customized shoe by using B-spline modeling and Multi-Layer Neural Networks techniques.

Machine learning and neural networks have also been used in the work [32-33]. Wang [35] proposes using shoe last scans for diabetic foot selection. Booth, on the other hand, uses machine learning to predict shoe last measurements.

7.4.4. Modular design methods

An interesting aspect is the implementation of modeling using modules. In this method, some parts of the shoe last are customized for a particular style of shoe, while others use the customer's anthropometric data to make ergonomic adjustments.

The method in the work conducted by Mishra et al. [39] essentially accounts for the shape of the customer's foot and the principles of three-dimensional ease allowance in a virtual environment. The design process begins with the creation of a graphic model of the shoe last using morphological contours of the foot. Both 3D models of the lower and upper surface of the personalized shoe last were created. Then a global 3D model of the personalized shoe insole was created, which consisted of the developed 3D models of the lower surface, 3D models of the rear part of the upper surface, and 3D models of the front part of the upper surface.

An extremely important solution in the field of shoe last customization is the modeling attempt proposed by Chertenko et al [41]. Their work proposed the method of modeling the ergonomic shape of a shoe last of the necessary style by combining two parts of different lasts using the surface and hybrid modeling features. The first paper Villarreal-Calva et al. [35] discusses the universal characteristics of the method and presents mathematical issues, while the second paper by Chertenko and Booth [42] presents the process of creating a high-heeled shoe prototype. The basis of the shoe last's shape is its back part (from the heel to the ball zone), which is constructed based on foot measurements obtained from anthropometric studies. These measurements are given as averages for a specific population group, considering biomechanical, physiological, and other characteristics.

7.4.5. Methods using Matlab software

MATLAB is a programming and numerical computing platform used by millions of engineers and scientists to analyze data, develop algorithms, and create models. The use of this software has also been used to customize shoe lasts.

This research has led to the virtual changes of the shoe last, anthropometric parameter and images have been achieved with the representation in section of the shoe last changed from the original shoe last by FEM method (Finite element method) in MATLAB environment.

Another example of Matlab usage is the use of a shoe last modeling automation algorithm [37]. This MATLAB algorithm can reconstruct the foot model to smooth the surface texture and rearrange the vertices of the three-dimensional (3D) model for easier calculation of dimensions. It can also locate the makers at the first and fifth metatarsophalangeal joints automatically for a more accurate design of the last shoe.

7.4.6. Other methods

Another example of the use of anthropometric data in the shoe modeling process is the work by Lypskyi et al. [47] and Skidan et al. [41], which explains the mathematical relationship between shoe last curves. Skidan et al [41] proposed the use of curves of Bezier curves to develop an analytical model. Lypskyi et al. [47] developed a mathematical model of the geometric modification of the complex three-dimensional shape of the sole of the shoe, which is performed during the design process. In this way, scientifically grounded parameters, built on the anthropometric structure of the foot, served as the basis for the algorithm and mathematical model of shoe sole modification.

One method of using anthropometric data is to create a shoe last gradation system through direct construction based on a virtual middle foot [40]. Based on standard feet, the method of morphological gradation was derived. To obtain parametric foot models, standard-sized polygonal models are reverse-engineered into spline models, thus allowing further processing by the 3D CAD system.

7.5. RQ5: Which scientific studies consider factors related to changes in foot dimensions resulting from loading, fatigue, and external factors?

In the works analyzed, there was no clear answer to the research question - perhaps this is related to the different subject matter to that proposed during the search strategy. Nevertheless, there is no information on how to consider the influence of the factors included in the research question on the process of footwear customization. As is known, the factors affecting changes in the length of the foot are natural and occur regardless of the type of footwear and use. No solution has been found that links the change in foot length to independent factors. Several studies attempt to solve this problem and take these parameters into account to some extent, but their reproducibility is not satisfactory.

Therefore, answering this research question would require an additional review of research and literature on the effects of foot fatigue from walking or standing on changing the basic dimensional characteristics of the foot. However, this topic is not developed in shoe last research, probably because the very idea of shoe last customization is extremely complex and additional parameters could complicate the procedure even more. The paper by Mishra et al. [39] introduces a shoe adjustment considering clearances that will account for changes in the shape of the foot during walking - however, this only applies to one case and no information is included on how the influence of external factors affects the model and clearances. A big plus of this is that the 3D model is free to adjust the geometry and consider the morphology of the foot significantly, compared to other works where only standardized or population data were relied on.

Another aspect, Amza et al. [30] discuss in the paper is the impact of changing parameters when creating a shoe last model. In their work, the authors propose a several-step algorithm to model a shoe last based on data taken from 3D scanning and matching it with reference shoe last models (standardized models). As it turned out, the more steps of the algorithm, the greater the dimensional changes in the designed model. Each modification of one shoe last parameter generates a change in the dimensions of the other parameters. The presence of the designer, who must control the change of parameters, is indispensable in this process. As you can see, such a solution is not able to automate the process completely.

7.6. *RQ6*: *What are the strengths, limitations, and weaknesses of the study*?

One of the main aspects of a systematic review is to analyze the strengths and weaknesses of each study. By considering the limitations or weaknesses, future attempts can be made to explore more contemporary approaches to shoe last design This topic has a lot of room for research, and constant changes in technology allow for better and better innovations. The following data is presented in tabular form for better readability of the text (Table 5).

Table 5 A comparison of the strengths, weaknesses, and limitations of the articles included in the systematic literature review

No.	Strengths	Weaknesses	Limitations
A01	1. 9-step shoe last modification algorithm. 2. using the Free Form. Deformation technique. 3. incorporate the tightening into the shoe last model. 4. comparative method of constructing the shoe last model: empirical and matrix. 5. optimization of reference points due to the occurring deformation of the foot. 6. validation of results.	1. no automation of the construction process. 2. no parameterization of the model. 3. Each modification of a model parameter results in changes in other parameters.	1. using a specialized scanner. 2. using dedicated software to determine reference points.
A02	1. model parameterization method. 2. 3D printing technique to produce prototypes.	1. the client's foot is not scanned, but the last model must be scanned. 2. no defined research groups. 3. factors related to foot deformation were not considered.	1. the diversified morphological structure of the feet was not considered when constructing the parameterized splines.
A03	1. combination of three machine learning techniques for shoe last design 2. validation of results.	1. Large errors in the last design were seen above the ankle and near the toes.	1. Errors in measurements resulted from not considering the style of shoes. 2. too few participants in the study.

Table 5 A comparison of the strengths, weaknesses, and limitations of the articles included in the systematic literature review (continued)

No.	Strengths	Weaknesses	Limitations
A04	1. analysis and implementation of several iterative trials highlighting the possibility of modifying the anthropometric parameters of virtual shoe last by converting them into linear sizes. 2. using the finite element technique for the shoe last optimization process.	1. no application of the method in the case of a selected population group. 2. lack of interpretation of changes in shoe last parameters about feet with dysfunctions.	1. using a commercial program.
A05	1. The method of modeling the ergonomic shape of a shoe last of the necessary style by combining two parts of different lasts using the surface and hybrid modeling features 2. 5-step customization algorithm 3. adapting the design last to diverse types of footwear	1. use of shoe last from the database - no data on feet with dysfunctions. 2. study population is not defined.	1. use of commercial software. 2. lack of smooth transition between the parts of the last.
A06	1. method of designing orthopedic shoes with clubfoot problem. 2. customization applied only from the patient's foot data (3D scanning). 3. the possibility of performing the method for other foot dysfunctions. 4. pain reduction and better functioning of the study participant thanks to the personalization of lasts.	1. focus on dysfunctional feet, no method is described for the non-disabled population.	1. method tested on 1 participant only.
A07	1. application of the foot model orientation algorithm concerning the axis of the coordinate system. 2. use of B-spline curves for shoe last modeling – 9-step algorithm.	1. no information on data validation. 2. no defined research population.	1. each pattern requires an optimization process - no standardization of algorithms.
A08	1. precise description of the research group. 2. clearly defined purpose of research. 3. automation of the process. 4. the use of neural networks for the classification process. 5. Detailed process algorithm. 6. The study uses the expertise of independent specialists. 7. defining the fitness function for the diabetic foot.	1. time-consuming algorithm.	1. too few shoe lasts were tested for the learning algorithm.
A09	1. using an algorithm to create a mathematical model identifying the reference point of the scanned foot. 2. the solution can be used to create lasts for normal shoes and the solution has been implemented for the design of cycling shoes. 3. 6-step shoe last model customization algorithm. 4. validation of test results.	1. no process automation method.	1. errors related to the boundary function - causing distortions or gaps in the model mesh.

Table 5 A comparison of the strengths, weaknesses, and limitations of the articles included in the systematic literature review (continued)

No.	Strengths	Weaknesses	Limitations
A10	1. automation of the foot model orientation algorithm according to the axis of the coordinate system. 2. an algorithm modeling shoe last circumferences based on data obtained after the shoe last scanning. 3. validation of created algorithms. 4. approach to the mass shoe-last customization process.	1. no research group defined. 2. no information on customization for the foot with dysfunction.	1. limitations resulting from the methodology of neural networks.
A11	1. Shoe last modeling based on three surface models. 2. surface analysis of scanned shoe last and cross sections to obtain a customized last. 3. consider the dimensional allowances of the last, which give comfort when using the footwear. 4. analysis of the literature and discussion of the morphological parameters of the foot.	1. Few issues related to the misalignment of curves when designing the top of the shoe last.	1. no information on the possibility of automating the process of mass personalization of shoe last.
A12	1. parameterization of the design process. 2. use of surface modeling. 3. include the variable height of the heel. 4. Position the model relative to the central coordinate system.	1. using ready-made models from databases. 2. no assumptions for the automation of the customization process.	1. occurrence of shading of the heel surface during scanning. 2. determination of differences for dimensions relevant to the design process and 3D coordinates.
A13	1. method of shoe last contour profiling using B-spline curves. 2. mathematical assumptions of the curve design method. 3. analytical solutions for modeling the longitudinal contours of the last.	1. study group not included. 2. no validation of results.	1. no information about the possibility of using the last design algorithm for diverse types of footwear.
A14	1. modeling based on the modular construction of the shoe last. 2. using a digital database of shoe lasts as reference models. 3. the front part of the shoe last obtained from reference data. 4. consideration of ergonomic factors.	1. lack of diversity in the research group. 2. consider the influence of the parameters of use, fatigue, and load on the design of the insole.	1. Problem related to the assessment of the comfort of using footwear. 2. shoe last milling method. 3. footwear design based on imported shoe inserts (no domestic production).
A15	1. a method using a comparison of the construction of the last by an experienced shoemaker and a designer using computer-aided algorithms. 2. the use of CNC machines to produce prototypes and footwear. 3. reducing the time associated with the production of shoes (optimization of modeling and shoe manufacturing methods). 4. the method describes surface modeling using curves.	1. described algorithm related to shoe-last modeling for diabetic foot only. 2. lack of full customer satisfaction in completed projects.	1. in some cases, uneven positioning of curves, requires constant control of the project by the designer.

Table 5 A comparison of the strengths, weaknesses, and limitations of the articles included in the systematic literature review (continued)

No.	Strengths	Weaknesses	Limitations
A16	1. automation of the process of forming the longitudinal profile of the shoe last. 2. methodology based on shoe last parameters, which have been defined as primary, secondary, and tertiary. 3. the use of logical dependencies of parameters to make a model of the shoe last. 4. noticeable reduction in the last design time. 5. data validation.	1. based solely on anthropometric data	1. lack of diversity in the research group
A17	1. parameterization of the model based on the model of the sole. 2. automation of the customization process. 3. Consider dimensional and geometric constraints in the parameterized model. 4. identifying reference points and dividing the sole into geometric parts to facilitate modeling. 5. method validation.	1. no defined research group and no qualitative study or ergonomic parameters.	1. model lasts are differentiated by the style of the shoe (this applies to parts of the toes and the heel). 2. the design concerns the upper of the shoe.
A18	1. Comparison of results concerning feet without dysfunction. 2. Using knowledge from previously conducted studies. 3. analytical explanation of the shoe last customization process.	1. customization process as the creation of new averaged shoe last models for the diabetic foot.	1. Shoe last modeling based exclusively on French shoe numbering.
A19	1. A mathematical model of geometric transformations of the shoe last model in the process of customization. 2. The study considers the influence of biomechanical, and physiological factors.	1. too small a research group.	1. no analysis of the customization of the various parts of the shoe last. 2. no information on the impact of the parameters tested on the construction or technological parameters of footwear.

8. Conclusions

Nowadays, research on the design of customized shoe-lasts is being conducted around the world. And their methodology covers various aspects. The use and advancement of technology make the traditional (manual) methods of creating shoes and shoe-lasts old-fashioned. Scanning and surface and curve-based modeling methods are becoming faster and more accurate. In many cases, shoe-lasts for clients with foot dysfunctions are being customized. It should also be remembered that mass customization should be spread increasingly because ill-fitting shoes can cause more losses in our lives than benefits.

A systematic review of the literature reviewed research in the field of shoe sole personalization. Nineteen articles that corresponded to the scope of the work were included. The literature items were selected based on exclusion and inclusion criteria - there were a total of ten (five - exclusions, five - inclusions). A total of six research questions were asked to classify the selected studies and to clearly include the spectrum of studies conducted. The study also included a description of the differences between the foot and the shoe last. In addition, a qualitative analysis consisting of seven criteria was carried out, which considered both the methodological parameters of the works and the general characteristics. The review of works assumed a research analysis of works from 2018 to 2022.

The first aspect of further research is to find an algorithm related to the unification of measurements taken during foot scanning - because none of the publications has solved the issue of normalization or unification of measurements. This is an important issue, but most often neglected in this type of solutions. Another research gap that has been found is the issue of automation of the design process. The construction process still largely depends on the designer and his experience - there is no program that would automatically transform input data about the shape and dimensions of the foot directly into a ready model of the hoof - this is an extremely difficult and complicated matter, but with the use of neural networks or machine learning is the ability to test the above hypotheses.

Conflicts of Interest

The authors declare no conflict of interest.

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