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Effect of Symmetry on Structures of Epicyclic Gear Trains

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Abstract

An epicyclic gear train (EGT) can be denoted by a graph and this graph in turn represented by an adjaceny matrix. Hamming matrix is used to detect isomorphism and measure the structural aspect of symmetry in EGTs. More symmetry in EGTs results in higher number of structural arrangements as compared to EGTs with no or less symmetry.

Keywords: epicyclic gear trains, adjaceny matrix, degree of freedom, isomorphism, symmetry, Hamming matrix.

1. Introduction

Synthesis and analysis are two important phases of machine design. Synthesis is a conceptual phase and is an important stage during the design process. Enumeration of mechanisms and epicyclic gear trains is achieved by systematic procedure. Graph theory is widely used by various researches [1-9] for synthesis of kinematic structures of EGTs with a given degree of freedom (DOF) and given number of links. In the synthesis of EGTs, detection of isomorphism in EGTs with given DOF and number of links during the generation process are essential. Various researchers used different approaches for both generation of EGTs and checking for isomorphism among the generated EGTs. Each approach has its own merits and demerits. During the synthesis of EGTs, one of the isomorphic chains (or graphs) is selected at random to generate EGTs with higher number of links.

There are many applications with EGTs like in wind turbine which need drives and overdrives that amplify the speed of turbine shaft and provide high speed at generator shaft, in gear boxes with twelve speed ratios in Volvo buses and differentials in the automobiles, gas turbine engines, in mechanisms including robots to transmit specified motion and/or torque between two or more shafts and also in helicopter transmissions, machine tool gear boxes etc. Gear trains constitute one of the best mechanisms, because gear trains represent a high level of engineering achievement besides rolling contact bearings.

A simple EGT (in Fig. 1) consists of one central gear called sun gear ‘3’ and one or more planet gears ‘2’ that rotate in mesh with the sun gear and revolve around the central sun gear on pin(s) attached to another element known as arm or planet carrier ‘1’. The drum of the element with internal teeth is known as annular wheel denoted by ‘4’. Consider a four link EGT shown in Fig. 1 and the corresponding graph representation [1-4] is shown in Fig. 2. This EGT is the one of the six 4-link EGTs those are generated from the basic 3-link EGT. Out of these six, only two EGTs are non-isomorphic. The remaining four EGTs are isomorphic to one of these two EGTs. In general out of the two sets of six 4-link EGTs, two non-isomorphic EGTs are selected at random, discarding the remaining four. Similar method is adopted while generating EGTs with five or six and higher number of links.

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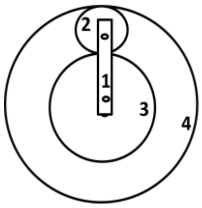


Fig. 1. Schematic representation

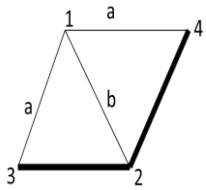


Fig. 2 Graphical eepresentation

There is no basis for selecting one and discarding remaining from a set of isomorphic EGTs of given number of links and DOF. It has been suggested [5] that there should be a definite reason like structural aspect or another of EGTs in selecting a particular chain out of a set of isomorphic chains for further generation. Symmetry in EGTs is one such characteristic. Symmetry is defined [5] as identical location of identical links in a graph. This fact is demonstrated with suitable examples. However other important benefits of symmetry are not addressed by them. In this work, an attempt is made to bring out the other effects of structural aspect of symmetry in EGTs.

In a graph of an EGT, the links are vertices and kinematic pairs are edges. Further turning pairs are represented by thin lines and thick lines are used to denote the gear pairs. Also the position of the axis for each shaft that carries an element in any EGT is denoted by a level. As shown in Fig. 2 the turning Pair 1-2 is at level b and the other turning pair 1-3 is at level ‘a’ and for the turning edge 1-4 is at level ‘a’. The graph of two non-isomorphic 4-link EGTs out of the six 4-link EGTs generated from the graph of a 3-link EGT is shown in Fig. 2 and Fig. 3.

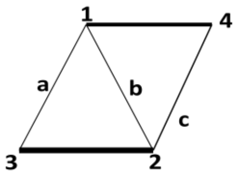


Fig. 3 Four link EGT

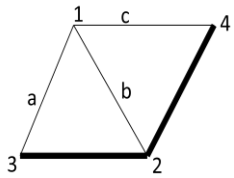


Fig. 4 Four link EGT

For the EGT shown in Fig. 4, the levels assigned to the three turning pairs i.e. level of pair 1-2 is b, level of pair 1-3 is ‘a’ and level of pair 1-4 is at a level ‘c’. The rules for identifying the levels of turning pairs are defined by L.W.Tsai [6] and Ravi Shankar and Mruthyunjaya [7]. However the level of pair 1-4 of graph in Fig. 2 can also be ‘c’ as shown in Fig. 4. The functional diagrams for these three EGTs shown in Figs. 2, 3 and 4 are shown in Fig. 5, 6 and 7 respectively.

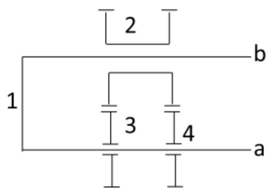


Fig. 5 Functional representation of Fig. 2

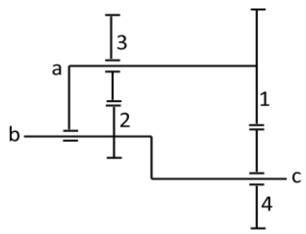


Fig. 6 Functional representation of Fig. 3

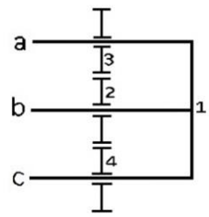


Fig. 7 Functional representation of Fig. 4

Fig. 5 is the Levai notation of the EGT shown in Fig. 2. If the level of the turning pair edge 1- 4 in the Fig. 3 can be either ‘a’ or ‘c’. The Levai notation for these two cases is different because the axis of rotation is different. Thus the Levai notation of the same EGT with turning pair edge at level ‘c’ can be represented as shown in Fig. 7; one can conclude that by changing the level of a turning pair for an EGT the structural diagram of the same EGT changes. Fig. 6 is the Levai notation of the EGT shown in Fig. 3.

2. Symmetry

Symmetry is identical location of identical elements in an EGT. From the Fig. 7, the links (gears) 3 and 4 are symmetrically placed about links 1 and 2 and obviously links 3 and 4 are identical links. Hence, the EGT in Fig. 5 is having symmetry. Further the EGT shown in Fig. 6 (or from the graph in Fig. 3) is not having any identical links. That is this EGT is not having any symmetry. This structural aspect can be measured at the conceptual stage without actually arranging the gear elements in an EGT from the Hamming matrix of the EGT.

3. Hamming Matrix

A graph of an EGT can be converted to a symmetric matrix called link–link Adjaceny matrix (A) on the lines similar to the rules given by Uicker and Raicu [4] for kinematic chains (KC) and modified by L. W. Tsai [6] and A. C. Rao [8] to suit the EGTs. Adjaceny Matrix of an EGT is given by Eq. 1

$$Adjacency\ matrix, A = \begin{cases} 1 \text{if link i is connected to link j by a turnming pair} \\ 2 \text{ if link i is connected to link j by a gear pair} \\ 0 \text{ if link i is not conncted to link j or when i} = \text{j} \end{cases} \tag{1}$$

and a_{ii}=0. Also, the rules for forming the symmetric Hamming matrix from an Adjaceny matrix of EGT are given by A. C. Roa [8] shown in Eq. 2.

$$h_{ij} = \begin{cases} \sum_{k=1}^{n} a_{ik} + a_{jk} \quad if\ a_{ik} \neq a_{jk} \\ 0\ if\ a_{ik} = a_{jk}\ ; \quad h_{ii} = 0; \end{cases} \tag{2}$$

As an example, consider the graph in Fig. 2 and its adjaceny and Hamming matrices are as follows:

$$\text{Adjaceny matrix for the EGT shown in Fig. } 2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\text{Hamming matrix for the EGT shown in Fig. } 2 = \begin{bmatrix} 0 & 8 & 6 & 6 \\ 8 & 0 & 6 & 6 \\ 6 & 6 & 0 & 0 \\ 6 & 6 & 0 & 0 \end{bmatrix} \tag{4}$$

Each row or column in a Hamming Matrix represents a link in the EGT and sum of all the elements in a row (or column) of a Hamming Matrix is the Hamming value of the link. If Hamming values of two links are the same, then the two links are said to be identical. Then from the above Hamming Matrix link 1 and 2 are identical and its Hamming value is equal to 20. Also, link 3 and 4 are identical and its Hamming value is equal to 12. Further, in first row the third and fourth links are same and is equal to 6. Hence, links 3 and 4 are symmetric about links 1 and 2. Also, it is clear from the Hamming matrix that link 3 and 4 are symmetric about link 2 and links1 and 2 are symmetric about links 3 and 4.The adjaceny and Hamming matrices for the EGT in Fig. 3 are

$$\text{Adjaceny matrix for the EGT shown in Fig. } 3 = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \tag{5}$$

$$\text{Hamming matrix for the EGT shown in Fig. } 3 = \begin{bmatrix} 0 & 8 & 7 & 5 \\ 8 & 0 & 5 & 7 \\ 7 & 5 & 0 & 6 \\ 5 & 7 & 6 & 0 \end{bmatrix} \tag{6}$$

From the above Hamming Matrix, links 1 and 2 are identical and links 3 and 4 are identical. However, the first row represents the link 1, Hamming values for links 3 and 4 are not the same. Hence, the links 3 and 4 are not symmetric about link 1. Same is the reason with other links. Hence even though there are two pairs of identical links no symmetry exists in the EGT which is evident from the structural diagram in Fig. 6. Thus, the aspect of symmetry can be determined from the Hamming matrix without actually drawing the structural diagram.

4. Effect of Symmetry on the Synthesis of EGTs

Structural arrangements or list of levels can be identified from the fundamental rules [6][9][10]. Two non-isomorphic 4–link EGTs shown in Figs. 2 and 3 an EGT shown in Fig. 2 is symmetric and two structural arrangements are possible from the graph by varying the levels as shown in Figs. 5 and 7. Whereas, the EGT shown in Fig. 3 have no symmetry [2], hence only one structural arrangement shown in Fig. 6 is possible. To further impress on this argument, consider the two non-isomorphic 5–link EGTs shown in Figs. 8 and 9.

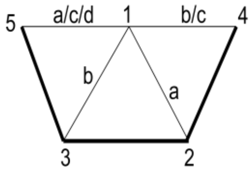


Fig. 8 Five link EGT

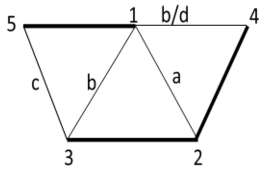


Fig. 9 Five link EGT

From the adjaceny and Hamming matrices of the above two EGTs, it is known that EGT in Fig. 8 is having high symmetry and EGT in Fig. 9 is having no symmetry. Also the various possible ways of assigning levels are indicated in the two Figs. 8 and 9. Thus five structural arrangements are possible for EGT in Fig. 8, whereas EGT in Fig. 9 has only two possible arrangements. They are listed in Table 1.

Table 1 Assigning levels to the EGTs shown in Figs. 8 and 9

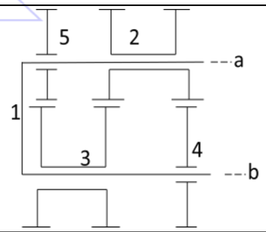
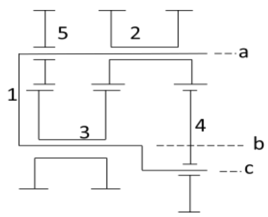
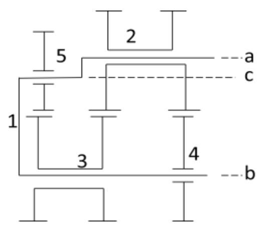
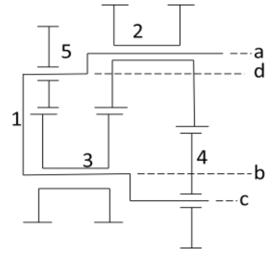
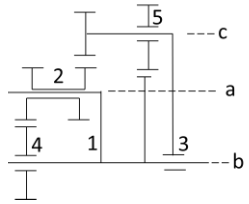
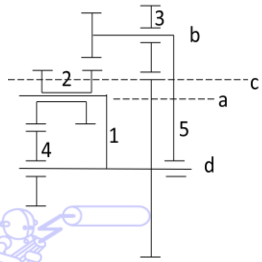
Graph	List of levels	Structural arrangement
EGT in Fig. 8 List of levels: total 5 (12,15)(13,14) (12,15)(13)(14) (12)(13)(14,15) (12)(13,14)(15) (12)(13)(14)(15)	(12,15)-level ‘a’ (13,14)- level ‘b’	
	(12,15)-level ‘a’ (13)- level ‘b’ (14)-level ‘c’	
	(12)-level ‘a’ (13,14)-level ‘b’ (15)-level ‘c’	

Table 1 Assigning levels to the EGTs shown in Figs. 8 and 9 (continue)

Graph	List of levels	Structural arrangement
EGT in Fig. :8 List of levels: total 5 (12,15)(13,14) (12,15)(13)(14) (12)(13)(14,15) (12)(13,14)(15) (12)(13)(14)(15)	(12)-level ‘a’ (13)-level ‘b’ (14)-level ‘c’ (15)-level ‘d’	
EGT in Fig. 9 List of levels: Total 2 (12,15)(13)(24) (12)(13)(24)(15)	(12)-level ‘a’ (13,14)-level ‘b’ (35)-level ‘c’	
	(12)-level ‘a’ (13)-level ‘b’ (35)-level ‘c’ (14)-level ‘d’	

Consider two EGTs with four gear pairs, shown in Figs. 10 and 11. From the adjaceny and Hamming matrices of these EGTS, it can be concluded that the EGT shown in Fig. 10 is a non-symmetric EGT and EGT shown in Fig. 11 is a symmetric EGT.

Table 2 Levels of six-link EGTs

Graph	List of levels	Number of possible levels
EGT in Fig. 10	(12)(13,35)(24)(26) (12,26)(13,35)(24)	Total=2 Different possible levels.
EGT in Fig. 11	(12)(13)(14)(15)(16),(12)(13)(14,15,16) 12)(13)(14,15)(16),(12)(13)(14,16)(15) (12)(13,14)(15)(16), (12)(13,14)(15,16) (12,16)(13,14,15),(12,16)(13,14)(15) (12,16)(13,15)(14),(12,16)(13)(14,15)	Total=10 Different possible levels.

The different levels possible for these fours EGTs are listed in Table 2. The EGT shown in Fig. 10 is having two possible structural arrangements and for the EGT in Fig. 11 have ten possible arrangements. The possible number of levels or arrangements for this EGT is only one. All the possible structural arrangements of EGTs shown in Fig. 10 and Fig. 11 are clearly given in Table-3 i.e. EGT in Fig. 10 has two and EGT in Fig. 11 has ten possible arrangements.

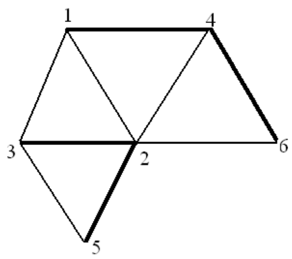


Fig. 10 EGT with six links

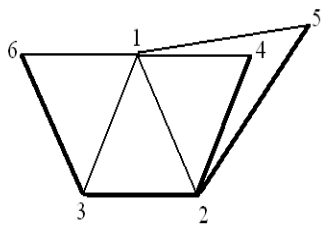


Fig. 11 EGT with four gear pairs

Table 3 Functional representation of EGTs gave in Table 2

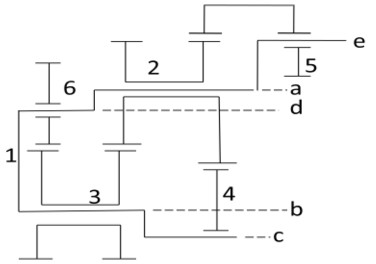
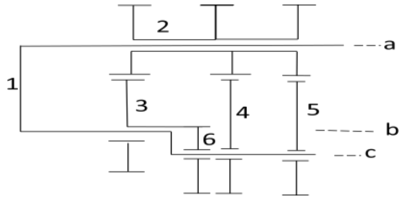
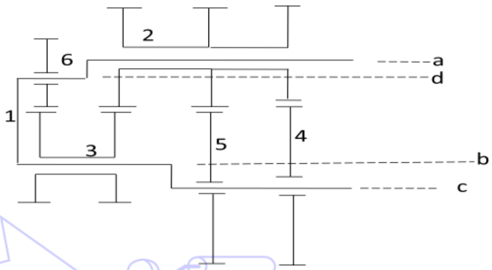
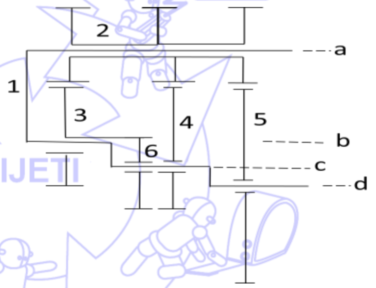
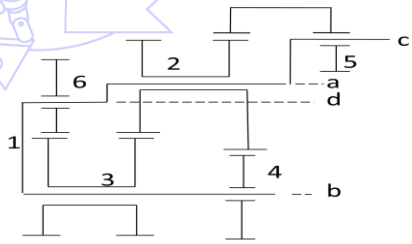
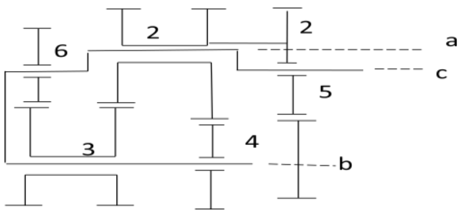
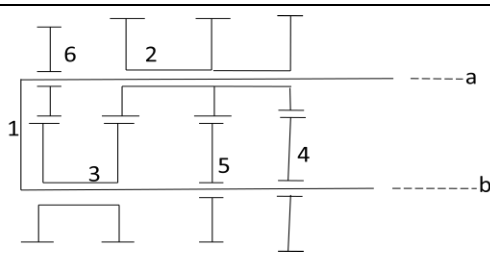
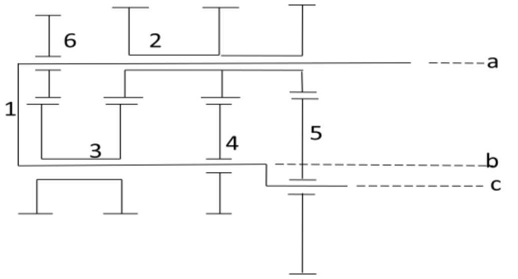
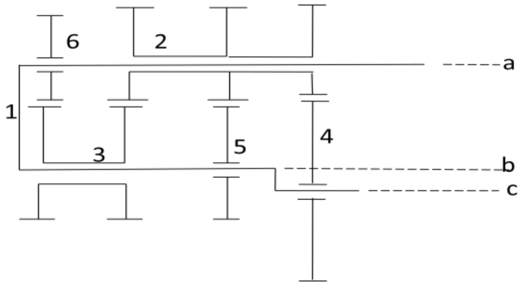
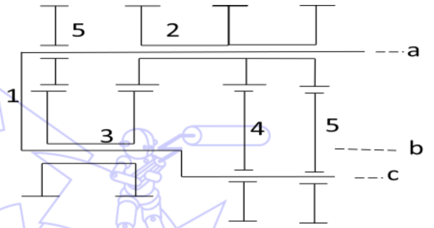
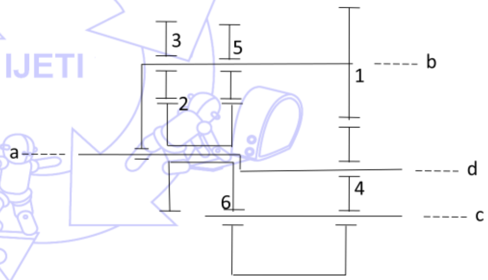
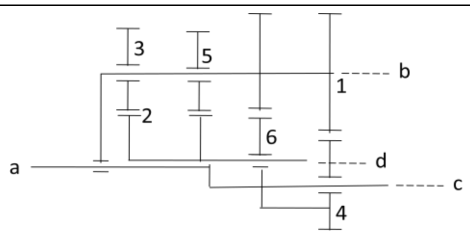
Graph	S.No	Possible level	Structural arrangements
	1	(12)(13)(14)(15)(16)	
	2	(12)(13)(14,15,16)	
	3	(12)(13)(14,15)(16)	
EGT in Fig. 11	4	(12)(13)(14,16)(15)	
	5	(12)(13,14)(15)(16)	
	6	(12)(13,14)(15,16)	
	7	(12,16)(13,14,15)	

Table 3 Functional representation of EGTs gave in Table 2 (continue)

Graph	S.No	Possible level	Structural arrangements
EGT in Fig. 11	8	(12,16)(13,14)(15)	
	9	(12,16)(13,15)(14)	
	10	(12,16)(13)(14,15)	
EGT in Fig. 10	1	(12)(13,35)(24)(26)	
	2	(12,26)(13,35)(24)	

As another example consider the three non–isomorphic 7–link EGTs shown in Figs. 12, 13 and 14. All the three seven-link EGTs are tested for isomorphism in links and symmetrical EGTs are identified and the possible arrangements of its structures with different levels are identified in each EGT. The Hamming matrices of all the seven-link EGTs shown in Figs. 12, 13 and 14 are given below. From the adjaceny and Hamming matrices of the EGTs shown in Fig. 12, no link is identical with any other link hence no symmetry exists in this EGT.

$$\text{Hamming matrix} = \begin{bmatrix} 00\ 10\ 06\ 10\ 06\ 05\ 06 \\ 10\ 00\ 08\ 12\ 12\ 07\ 06 \\ 06\ 08\ 00\ 04\ 04\ 07\ 06 \\ 10\ 12\ 04\ 00\ 08\ 11\ 08 \\ 06\ 12\ 04\ 08\ 00\ 07\ 08 \\ 05\ 07\ 07\ 11\ 07\ 00\ 07 \\ 06\ 06\ 06\ 08\ 08\ 07\ 00 \end{bmatrix} \tag{6}$$

$$\text{Hamming matrix} = \begin{bmatrix} 00 & 13 & 09 & 09 & 07 & 07 & 09 \\ 13 & 00 & 08 & 12 & 10 & 10 & 08 \\ 09 & 08 & 00 & 08 & 06 & 06 & 00 \\ 09 & 12 & 08 & 00 & 10 & 02 & 08 \\ 07 & 10 & 06 & 10 & 00 & 08 & 06 \\ 07 & 10 & 06 & 02 & 08 & 00 & 06 \\ 09 & 08 & 00 & 08 & 06 & 06 & 00 \end{bmatrix} \tag{7}$$

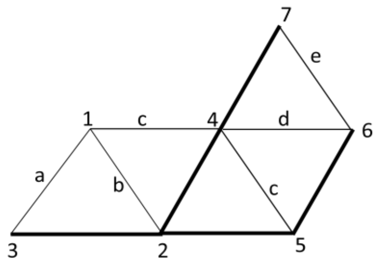


Fig. 12 Seven link EGT

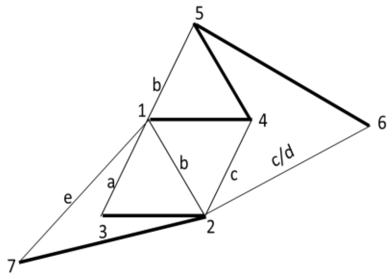


Fig. 13 Seven link EGT

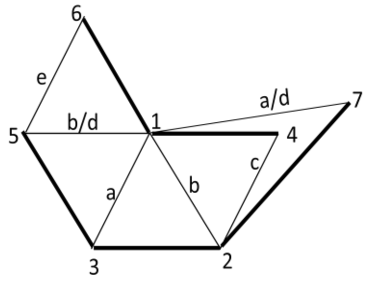


Fig. 14 Seven link EGT

Consider another EGT as shown in Fig. 13. From the Hamming matrix links 3 and 7 are having identical Hamming strings and links 1, 2, 4, 5 and 6 are symmetric with respect to links 3 and 7. Hence it is a symmetric EGT. From this EGT three different levels or arrangements are possible.

$$\text{Hamming matrix} = \begin{bmatrix} 00 & 14 & 13 & 09 & 12 & 09 & 11 \\ 14 & 00 & 09 & 09 & 04 & 09 & 07 \\ 13 & 09 & 00 & 08 & 07 & 08 & 02 \\ 09 & 09 & 08 & 00 & 07 & 02 & 06 \\ 12 & 04 & 07 & 07 & 00 & 07 & 05 \\ 09 & 09 & 08 & 02 & 07 & 00 & 06 \\ 11 & 07 & 02 & 06 & 05 & 06 & 00 \end{bmatrix} \tag{8}$$

Consider another EGT shown in Fig.14. The link Hamming strings of links 4 and 6 are identical from the given Hamming matrix. Hence, the links 4 and 6 are isomorphic with each other; links 1, 2, 3, 5 and 7 are symmetric with respect to links 4 and 6.

Table 4 Levels of seven link EGTs

Graph	List of levels	
EGT in Fig. 12	(12)(13)(14,45)(46)(67)	Total=1 possible level.
EGT in Fig. 13	(12,15)(13)(17)(24,26), (12,15)(13)(17)(24)(26) (12,15)(13,17)(24,26)	Total=3 Different Possible levels.
EGT in Fig. 14	(12,15)(13,17)(24)(56), (12)(15)(13)(17)(24)(56) (12)(15)(13,17)(24)(56), (12,15)(13)(17)(24)(56) (12)(13)(15,17)(24)(56)	Total=5 Different Possible levels.

Five links are symmetric with respect to two links 4 and 6 hence the EGT shown in Fig. 14 is highly symmetric. The Hamming string of links 4 and 6 are L4 = [9, 9, 8, 7, 6, 2, 0] = L6. The EGTs shown in Figs. 13 and 14 are symmetric EGTS while the EGT shown in Fig. 12 is a non-symmetric EGT. The different levels possible for these three EGTs are listed in table 4.

Thus, for the EGTs in Fig.13 and 14 three and five structural arrangements are possible and for the EGT in Fig. 12 has only one possible arrangement. Hence, it can be concluded that an EGT having more symmetry results in higher number of structural arrangements and EGT with lesser or no symmetry gives least number of arrangements.

5. Conclusion

Symmetry is important both from architectural beauty of the structure and generation of structures. Better balance can easily be achieved by having more symmetry in the EGT. Generation process is simplified if symmetry aspect is incorporated into synthesis process. The higher the number of symmetric pairs is in an EGT is, the greater is the symmetry. Symmetry plays a vital role from the aesthetics point of view and in the generation of geared kinematic chains. Knowing the symmetry in a graph of a GKC, the number of isomorphic graphs/structures enumerated can be reduced to a greater extent. Identifying isomorphism in structural synthesis of EGT is very important. However, while selecting one of the isomorphic graphs for further generation-structural aspects like symmetry should be taken into consideration. An EGT with higher symmetry results in more number of structural arrangements than an EGT with no symmetry. Symmetry gives better balancing of elements, forces, power distributions and leads to reduction in generation effort. For better balance, two or more planet gears are added to increase the number of forces to the existing gear trains. But the added planet gears do not affect the kinematic performance. Hence these additional planet gears are referred as idler gears; further aesthetics is also improved by structural symmetry. Instead of random selection of an EGT from a set of isomorphic EGTs for a given number of elements and DOF, it is advised to select one with higher symmetry. This is summarized with a good number of examples.

References

[1] R. Rajasri, Y. Rao, and A. Gupta, “Structural Aspects of Symmetry and Its Effects on Generation of Planetary Gear Trains,” 2011 International Conference on Mechanical and Aerospace Engineering, Mar. 2011, pp. 530-533.
[2] R. Rajasri, A. Gupta, and Y. Rao “Influence of structural aspects on the generation process in planetary gear trains,” International Journal of Scientific Research, vol. 3, no. 10, pp. 1018-1021, Oct. 2011.
[3] Z. Lévai, “Structure and analysis of planetary gear trains,” J. Mech, vol. 3, pp. 131-148, Aug. 1968.
[4] J. J. Uicker and A. Raicu, “A method for the identification and recognition of equivalence of kinematic chains,” Mech. Mach. Theory, vol. 10, no. 5, pp. 375-383, 1975.
[5] A. C. Rao and Y. V. D. Rao, “Symmetry in planetary gear trains,” Indian Journal of Engineering and Material Sciences, vol. 9, pp. 311-314, Oct. 2002.
[6] L. W. Tsai, “An application of the linkage characteristic polynomial to the topological synthesis of epicyclic gear trains,” ASME Journal of Mechanisms, Transactions and Automation in Design, vol. 109, no. 3, pp. 329-336, 1987.
[7] R. R. Shankar and T. S. Mruthyunjaya. ‘Computerized Synthesis of the Structure of General kinematic chains’, Mechanism and Machine Theory, vol. 20, no. 5, pp. 367-387, 1985.
[8] A. C. Rao, “Hamming number technique I: further applications,” Mech. Mach. Theory, vol. 32, no. 4, pp. 477-488, May 1997.
[9] A. C. Rao, “Comparison of linkage mechanisms,” Indian Journal of Technology, vol. 28, pp. 529-534, September 1990.
[10] G. Chatterjee and L. W. Tsai, “Enumeration of epicyclic-type automatic transmission gear trains,” SAE Transactions, pp. 1415-1426.