Dynamic Allocation Strategy for the Car Rental Industry with Multiple Rental Channels

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Abstract

This paper addresses the rental car allocation problem in which a car rental company rents its cars through various channels. This study proposes a constrained model based on the nested-allocation structure to allocate the rental cars to the car rental system to maximize the total rental revenues. The nested allocation strategy uses the concept of whether the number of sales accumulated to a certain level exceeds a threshold corresponding to that sharer. This approach allows the quota of low-profit channels to support the leasing demand of high-profit channels at any time, thereby increasing revenue. The proposed model can dynamically adjust the nested strategy as the dynamic programming model. However, it avoids the issues of ample storage space and long calculation time commonly encountered by the dynamic programming model. Computational results show that the proposed nested approach substantially outperforms the traditional exclusive allocation approach.

Keywords: car rental, constrained mathematical model, heuristic, multi-channel, nested allocation

1. Introduction

The car rental market has been increasing annually in several countries. Car rental has various purposes, such as business travel, leisure travel, outdoor travel, emergency travel, official vehicle, and international travel. The car rental market's revenue and number of users are expected to reach US$99.27 billion and 620 million people, respectively, in 2023. The compound annual growth rate of revenue from 2023 to 2027 is expected to be 2.99%. Based on this estimate, revenue and user numbers are expected to reach US$111.7 billion and 620 million people, respectively, by 2027 [1]. These show that the car rental market will still grow in the future. The car rental market continues to thrive due to many reasons. Renters do not have to pay a considerable amount of car purchases, car taxes, and maintenance costs. They do not have to drive long distances to reach tourist destinations. They can enjoy various car models and rental services provided by rental companies [2].

Car rental companies solicit customers extensively to attract different consumer groups. With the rapid development of technology, car rental companies can provide consumers with car rental services through multiple rental channels, such as travel agencies and professional leasing websites. Multichannel sales systems have been widely applied in many industries [3]. For example, airlines and restaurants sell to customers at different rates through various channels, including Expedia, Ctrip, Booking, and Skyscanner [4]. Hotels accept customer reservations through traditional counters and online agents [5]. These companies adopt multiple sales channels with different discount prices to sell a fixed amount of resources to maximize their revenues [4].

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Multirental channel systems are a win-win mechanism because diversified rental channels can help operators increase revenue by providing more sales opportunities. At the same time, they offer consumers the chance to obtain products through low-priced channels, reducing their spending [6]. However, improper allocation of car resources may cause an imbalance between supply and demand, and this imbalance results in inadequate rentable cars in some rental channels and excess rentable cars in others. In general, operators may expect that high-profit rental channels can rent most cars and may attempt to allocate more cars to the high-profit channels but fewer to the low-profit channels.

However, this strategy cannot guarantee high rental performance because car rental demand through high-profit channels may be much lower than the number of car resources allocated. Comparatively, rental demand through low-profit channels may be much higher than allocated car resources. In this case, many rental resources in higher-profit channels will be left idle, and the rental system will lose considerable rental chances in lower-profit channels. The total number of rental cars allocated to the rental station in the car rental industry usually stays the same in the short term. Therefore, operators must appropriately allocate cars to each channel to reduce the possibility of idle or insufficient rental cars [7].

The exclusive and nested allocation methods are two popular resource allocation methods. The exclusive allocation method divides the total resource into several quotas, assigning each quota to a specific sharer. Each sharer has different consumer groups. Each sharer’s allocated quota represents the maximum available resources for that sharer. The sum of the resources assigned to all sharers is precisely the total number of resources. This method has been used to solve various problems, such as personal distribution and budget allocation.

The exclusive allocation strategy has a less complicated calculation structure. However, the allocation quota of each sharer is independent; thus, the operation system cannot use the resources allocated to each sharer to support other sharers’ needs. Hence, some sharers utilizing this method may leave too many resources, while others may have insufficient resources. For example, a customer makes a rental request at a sharer who has no resources on hand. In this situation, even if some other sharers have less profit than the sharer who still has resources, the system has to reject the customer’s request because this higher-profit sharer has no resources to satisfy its customer’s request.

The nested allocation method is a crucial resource allocation technology in revenue management (RM). RM is a management method of selling products or services to customers at the right time and price to obtain the maximum expected revenue [7]. The application of this method has received excellent results in many industries, such as the airline industry, hotels, restaurants, and bars [8]. The nested allocation strategy does not set a limit for each sharer. Still, it uses whether the number of sales accumulated to a certain level exceeds a threshold corresponding to that sharer. This method’s structure based on cumulative sales across multiple channels allows holders to share their resources. It can reduce the possibility that some leased station resources are idle while others are insufficient [7].

A rental company may expect to dynamically adjust the car resources to each rental channel through a few nested thresholds based on the car inventory level at each rental station at different points in time. One way to develop dynamic nested allocation strategies is using dynamic programming models. However, dynamic programming algorithms face time and space complexity issues because they must store and compute many intermediate results, leading to exponential storage and computation time growth as the problem size increases [9].

Thus, this paper introduces a method that utilizes a mathematical programming model to establish a dynamic nested strategy for allocating rental cars to multichannel car rental systems to mitigate these challenges. The rest of this article is organized as follows: Section 2 provides a literature review. Section 3 describes relevant assumptions of the research problem and proposes a constrained integer linear programming model for the technical staff shift planning problem. Section 4 introduces the solution method. Section 5 uses numerical examples to illustrate the application of the model to the multi-channel car rental problem. Finally, Section 6 provides the conclusion of the paper.

2. Literature Review

The problem discussed in this paper belongs to the multi-rate and multi-channel resource allocation problems. The multi-rate resource allocation problem has been widely discussed in RM issues, especially in the airline and hotel industries. Problems associated with these systems usually involve multi-dimensional environments such as multi-rental stations, multi-day reservations, and multi-period reservations, and most of them are complex computational network RM problems. Oliveira et al. [10] reviewed car rental fleets' operation and RM issues, proposed a new integrated conceptual framework, and identified some research directions for future development. Many scholars have used the decomposition-based approximate solution to build controls for various capacities, such as the resource allocation problem of hotel room allocation for solving large-scale problems [11]. These methods are used to divide a large-scale network problem into several independent subproblems and have the disadvantage of disrupting the network effect of resource sharing among different products.

Kunnumkal and Topaloglu [12] used segment-based decomposition to solve airline RM problems involving customer choice. They first allocated the revenue from each trip to the segments covered by the trip. They combined the penalty value with the network effect, regarded the income distribution and penalty value as decision variables, and used the subgradient search method to solve the problem. Aslani et al. [13] proposed a decomposition method to solve the problem of hotel RM pricing. They considered out-of-stock and customer losses due to high-price strategies and developed techniques to estimate effective daily arrival rates. They also used daily arrival rates to break down network problems into several single-day problems. Li and Pang [14] constructed a discrete-time dynamic programming model to discuss the dynamic reservation control problem of RM for car rental businesses with a single car rental station.

Due to the computational difficulty, they proposed two decomposition-based heuristic methods to solve the problem. Fontem [15] constructs a dynamic programming model to explore a leasing and selling problem with car leasing and buying customers. As the accumulated rental trips increase, the sales price of a rental car decreases. This article establishes a threshold policy determining how long to lease a vehicle. Some studies have focused on measuring the impact of decomposition methods on network effects based on time and capacity bid prices to improve the performance of these methods [13]. Bidding price control is a widely adopted strategy in airline RM studies. The concept of this method is to set a threshold price for each resource (e.g., flight segment and hotel daily capacity).

Then, this method decides whether to accept or reject a request based on whether the revenue from the booking demand exceeds the total bid price of all resources consumed by the market. Kunnumkal and Topaloglu [12] formulated the total expected profit based on the bid price. They used a stochastic gradient to construct a random approximation algorithm to develop a bid price policy and reduce the computational burden. Pimentel et al. [16] compared the revenue-generating capabilities of the nested network and bid price methods in hotel RM. They found that the nested network method is better than the bidding method because the nested network method can increase revenue by an average of 6% in the worst case. Li et al. [17] built a stochastic dynamic model to explore the RM problem of a car rental network with different rental times and inventory liquidity. They proposed a Lagrangian relaxation method to solve the proposed model by decomposing the problem into multiple single-station and single-day problems.

Many workers developed heuristic approaches to solve the problem. Conejero et al. [18] built an integer linear model to explore the fleet allocation problem, which assumes that when the existing fleet cannot satisfy all accepted reservations, operators can lease vehicles from an external provider and treat them as the initial portion of the rental fleet. They developed a heuristic algorithm to solve this problem and minimize the number of cars subcontracted from the external supplier. Oliveira et al. [19] proposed a mathematical model to analyze a dynamic pricing and capacity decision-making problem for car rental companies. Due to the high computational complexity of this problem, they adopted a decomposition method that fuses genetic algorithms to encode the price decision in the solution search individuals of the genetic algorithm.

Later, Oliveira et al. [20] constructed a mathematical model in which demand is price-dependent to explore the number and type of fleets for a car rental company. They proposed a method based on genetic algorithms to solve the problem. They developed an approach based on genetic algorithms to solve the problem. Queiros and Oliveira [21] proposed a bi-objective model to explore a capacity pricing problem that incorporates environmental factors into car rentals. They developed an Epsilon-constraint approach to achieve the goal of a trade-off between profit and environmental impact. Jiang et al. [22] applied a genetic algorithm to optimize vehicle routing with infeasible routes to minimize losses and enhance customer satisfaction significantly. Chen et al. [23] developed a hybrid evolutionary algorithm to optimize the placement of signal transmission stations with different regional communication quality constraints.

The research mentioned above does not explore the issue of multichannel resource allocation. Schneider and Klabjan [24] proposed a single-item loss sales model in the context of two sales channels and analyzed the best conditions under known inventory control strategies. They analyzed three variants of three arrival process models. However, their models ignore the rationing strategy. Tian et al. [25] constructed a multi-period multichannel continuous-time model of time-dependent arrival rate. They used the channel control strategy and the decision to protect the threshold for all periods to maximize profit. They developed the threshold path control strategy based on the maximum principle.

According to the access control strategy, they found a unique optimal protection level at each period. Wang et al. [26] aimed at the RM problem with a single flight and multiple rate levels under various sales channels to confirm the benefits of RM in the sales channel. They constructed two mathematical models for cases without and with customer transfer of dynamic programming. Numerical example studies have found that airline revenue systems introducing channel allocation strategies can significantly increase revenue and reduce channel costs. The main reasons for the improvement are improved distribution between fare categories and reduced risk of lost revenue.

Managers of car rental companies should efficiently allocate car resources to each car rental channel to increase the utilization rate of the leased cars and reduce the possibility of a car rental operator shortage to increase revenues. The literature above indicates that developing multichannel leasing systems can increase leasing opportunities to improve revenue, using the nested allocation strategy can improve resource allocation efficiency, and applying the linear programming technique can improve the computational barriers of significant network problems.

Thus, research on multichannel car rental allocation with a nested allocation strategy is a valuable research topic. However, few studies have explored this issue. Therefore, the current study develops a linear programming model and applies a nested allocation approach to allocate the rental car resource to each car rental channel to maximize the total rental revenues over a planning horizon for multichannel car rental systems.

3. Model Description and Formulation

Consider that a car rental company plans to develop a *T*-period leasing strategy for its *M* rental stations. Each period can be one day, one hour, or two hours. The current number of available rental cars at rental station *i* is s_i. Consumers can rent a car at any rental station and return the rented car to any rental station. The record of a car rental includes the rental station, the rental channel, the return station, and the number of rental periods.

Let (t, i, j, k) represent the rental type where a car is rented at rental station *i* in period *t* for *k* periods and is returned to rental station *j* at the end of period $t + k - 1$. The company rents its cars through *E* channels. The revenue fee of rental type (t, i, j, k) through channel *e* is r_{tijk}^{e} with the condition that channel 1 has the lowest revenue, followed by channel 2, and so on. That is, $r_{tijk}^{1} < r_{tijk}^{2} \cdots < r_{tijk}^{E}$. Let a period be divided into *N* time slots, in which a maximum of one rental type appears. The probability that d requests for rental type (t, i, j, k) at channel e in any subperiod of t is denoted by p_{tijkd}^{e} with

$\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{d=1}^{D}p_{tijkd}^{e} \leq 1$ where K is the maximum rental period. This condition implies that demand at channel e for car rental type (t, i, j, k) in a subperiod of t is expected to be $\sum_{d=1}^{D} dp_{tijkd}^{e}$. Given that a period has N subperiods, the expected demand in the whole period of t is $d_{tijk}^{e} = N\sum_{d=1}^{D} dp_{tijkd}^{e}$.

For the model development, this paper assumes that the event of car returns occurs at the end of the expiration period. Suppose the current period is t; let b_{tik} represent the number of rental cars customers will return to rental station i after k periods starting from period t. At the end of each period, several cars will still be in leasing after the car return event. Denote O_t as the number of cars still rented out after the car return event at the end of period t. Let the symbol v_{ti} represent the car inventory at station i at the beginning of period t. Car inventory is related to whether sufficient cars are available for lease and will change over time due to car rental and return. Car rental at each station is affected by demand and the number of leasable cars at each channel. Regarding the nested allocation approach, let the symbol u_{ti}^{e} denote the nested rental threshold for channel e at rental station i in period t. The rental system determines whether to accept a rental request based on related threshold values. How to develop these values is illustrated as follows. The notation is summarized below in Table 1.

Table 1 Notation and variables

Notation			
E	Number of rental channels	S	Total number of rental cars
K	Maximum rental periods	T	Total number of planning periods
M	Total number of rental stations	N	Number of sub-periods in a period
s_i	Number of available cars at rental station i at the beginning of the first period		
O_{ik}	Number of vehicles customers will return to rental station i after $k-1$ periods starting from the first period with $O_{iK}=0$		
c_k^e	Unsatisfied cost per unit of the leasing car for k periods		
c_{ij}	Unit car transportation cost for rental station i to rental station j		
r_{tijk}^{e}	Rental price of the rental type (t, i, j, k) through channel e		
d_{tijk}^{e}	Expected demand for rental type (t, i, j, k) at channel e		
Variables			
v_{ti}	Number of rental cars at station i at the beginning of period t		
w_{ti}^{e}	Maximum rental limit for channel e at the rental station i in period t		
u_{ti}^{e}	Nested rental limit from the lowest revenue channels E to channel e at the rental station i in period t		
q_{tijk}^{e}	Expected number of rented rental type (t, i, j, k) in channel e		
b_{tik}	At the end of period t, the number of leased cars that will be returned to station i at the end of period $t+k-1$		
O_t	Number of cars in leasing after car return at the end of period t		

The structure of the mathematical model is to maximize the total net profit of total rental revenues minus total unsatisfied costs subject to the following constraints.

(1) Constraints for nested allocation structure: These constraints establish a nested allocation structure.

(2) Constraints for nested booking limit: These constraints ensure that the expected number of accepted rentals does not exceed the booking limit.

(3) Constraints for acceptable rental cars: These constraints ensure that the expected number of accepted rentals does not exceed demand.

(4) Equation constraints for the number of cars in leasing: These constraints express the number of cars customers will return to each station.

(5) Equation constraints for inventory level at rental stations: These equations express the car inventories at all stations at all periods.

(6) Equation constraints for keeping a total number of cars equal: These equations express that the number of cars stored in the rental station plus the number of cars leased out permanently equals the total number.

The following describes the objective function and each constraint formula.

Objective function:

The revenue of rental type (t, i, j, k) from channel *e* is the result of the number of cars rented out q_{tijk}^{e} times the rental fee r_{tijk}^{e} . Thus, by summing all possible rental types over all periods, the total revenue can be expressed as $\sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e}\, r_{tijk}^{e}$. By definition, the expected demand of rental type (t, i, j, k) from channel *e* is d_{tijk}^{e} and the expected number of accepted requests is q_{tijk}^{e}. It must be $q_{tijk}^{e} \leq d_{tijk}^{e}$ because the accepted request cannot be larger than the rental request. Thus, the expected unmet cost of rental type (t, i, j, k) from channel *e* is $\left(d_{tijk}^{e} - q_{tijk}^{e}\right)c_{k}^{e}$, and the overall unmet cost from all possible rental types over all channels can be expressed as $\sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} c_{k}^{e}\,(d_{tijk}^{e} - q_{tijk}^{e})$. Accordingly, the objective can be expressed by:

$$\text{Max PR} = \sum\nolimits_{t=1}^{T}\sum\nolimits_{e=1}^{E}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{M}\sum\nolimits_{k=1}^{K} q_{tijk}^{e} r_{tijk}^{e} - \sum\nolimits_{t=1}^{T}\sum\nolimits_{e=1}^{E}\sum\nolimits_{i=1}^{M}\sum\nolimits_{j=1}^{M}\sum\nolimits_{k=1}^{K} c_{k}^{e}\left(d_{tijk}^{e} - q_{tijk}^{e}\right) \tag{1}$$

Constraints for nested allocation structure:

Suppose v_{ti} cars are available at the start of period *t* in station *i*. Below, this paper first illustrates the exclusive and nested allocation approaches. In the exclusive allocation structure, the value of w_{ti}^{e} is exclusively used by channel *e*, and it is the maximum rental level of channel *e*. Thus, the relationship between w_{ti}^{e} and v_{ti} is given by:

$$\sum\nolimits_{e=1}^{E} w_{ti}^{e} = v_{ti}^{e},\ \forall i, t \tag{2}$$

In the nested allocation approach, this method sets a threshold to limit the number of leases for the highest income channel in each group of channels from the lowest income channel to the highest income channel instead of giving a dedicated number of cars to each distinct channel. Let the symbol u_{ti}^{e} represent the rental limit for channels 1 to *e*. The value of u_{ti}^{e} is the cumulative value of $w_{ti}^{e'}$ from e' = 1 to $e' = e$ and is expressed by:

$$u_{ti}^{e} = \sum\nolimits_{e'=1}^{e} w_{ti}^{e'},\ \forall e, i, t \tag{3}$$

Constraints for nested-booking limit:

The value of u_{ti}^{e} is used to decide whether to accept customers' car rental requests at station *i* through channel *e* in period *t*. The value of q_{tijk}^{e} is defined as the accepted number of rental types (t, i, j, k) at channel *e* in period *t*. The number of all possible rental types of (t, i, j, k) rented through channel 1 to channel *e* in period *t*, $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e}$ should be no larger than the value of u_{it}^{e}. Thus, constraint is required.

$$\sum\nolimits_{e=1}^{E}\sum\nolimits_{j=1}^{M}\sum\nolimits_{k=1}^{K} q_{tijk}^{e} \leq u_{ti}^{e},\ \forall i, t \tag{4}$$

Constraints for acceptable rental cars:

The accepted number of (t, i, j, k) rentals at channel *e*, that is, q_{tijk}^{e}, cannot be larger than the demand, that is, d_{tijk}^{e}. Thus, the following constraint is needed.

$$q_{tijk}^{e} \leq d_{tijk}^{e},\ \forall e, i, k, t \tag{5}$$

This model still needs equations to update the stock of rental cars at rental stations, cars in leasing, and rental cars to be returned over time, as well as constraints to ensure that the total number of cars remains unchanged.

Equation constraints for the number of cars in leasing:

Car inventory at the start of a particular period equals the number of car inventory at the start of the previous period minus car rental in the last period and the number of returned cars at the end of the prior period. Suppose that the current period is *t*. b_{tik} is used to represent the number of cars that will be returned to station i at the end of period $t + k - 1$. This value comes from two parts:

part (i) the number of cars rented out before period *t* and customers will return them to rental station *i* at the end of period $t + k - 1$.

part (ii) the number of cars rented out in period *t* that customers will return to rental station *i* at the end of period $t + k - 1$.

For period $t = 1$, the value of part (i) is the number of cars left at the start of period 1 of O_{ik}. The value of part (ii) is calculated as follows. The accepted rental requests of renting from station *j* and returning to station *i* for *k* periods from channel *e* will be returned at the end of period $t + k - 1$. Note that the expected number of these rental cars is defined by q_{1jik}^{e}. By summing all possible rental types of $(1, j, i, k)$ overall rental stations from all channels, the value of part (ii) can be expressed as $\sum_{e=1}^{E}\sum_{j=1}^{M} q_{1jik}^{e}$. Thus, $b_{1,ik}$ is given by:

$$b_{1,ik} = O_{ik} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{1ijk}^{e}, \ \forall i, k \tag{6}$$

For period $t > 1$, the value of part (i) of b_{tik} is the number of leased cars that customers will return to station i at the end of period $t + k - 1$, $b_{t-1,i,k+1}$. In addition, for $k < K$, the value of part (ii) of b_{tik} is the number of leased cars that customers will return to rental station *i* at the end of period $t + k - 1$, $\sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjik}^{e}$. Thus,

$$b_{tik} = b_{t-1,ik+1} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjik}^{e}, \ \forall t > 1, i, k < K \tag{7}$$

For period $t > 1$ and rental period $k = K$, among the cars in leasing in the previous period, the customers with the latest return period are those customers with the maximum number of lease periods *K* in period $t - 1$. However, these customers' return periods are at the end of $t + k - 2$. Thus, the value of part (i) of b_{tiK} is 0. For part (ii), only cars leased in period *t* with rental periods *K* will be returned at the end of period $t + K - 1$. The number of these rentals is $\sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjiK}^{e}$. Thus, b_{tiK} is given by:

$$b_{tiK} = \sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjiK}^{e}, \ \forall t > 1, i \tag{8}$$

The value of b_{tik} is denoted as the number of leased cars that will be returned to station *i* at the end of period $t + k - 1$. After the cars' return at the end of period *t*, b_{tik} for all i and $k > 1$ is still in leasing. Thus, when the maximum rental period is $K = 1$ given that all rented cars are returned to a certain station at the end of the same period, O_t is given by:

$$O_t = 0, \ \forall t, K = 1 \tag{9}$$

When the maximum rental period $K > 1$, the formula of O_t for $t < K$ is given by:

$$O_t = \sum_{i=1}^{M}\sum_{k=2}^{K} b_{tik}, \ \forall t, K > 1 \tag{10}$$

Equation constraints for car inventory level at rental stations:

Given that the initial car allocation at rental station *i* is set at s_i, the following constraint holds.

$$v_{1i} = s_i, \ \forall i \tag{11}$$

For station *i*, the total number of cars rented from rental station *i* through all channels is $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{ijkt}^{e}$. The car inventory at the start of period $t+1$ is the result of car inventory at the start of period *t*, that is, v_{it}, minus the cars rented out during period *t*, that is, $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{ijkt}^{e}$, plus the number of cars returned to station *i*, that is, b_{ti1}. Thus, $v_{i,t+1}$ is expressed by:

$$v_{t+1,i} = v_{ti} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e} + b_{ti1},\ \forall i, t > 1 \tag{12}$$

Equation constraints for keeping the total number of cars equal:

The number of cars allocated at stations *i* is s_i. let $S = \sum_{i=1}^{M} s_i$ be the total number of cars. Given that the total number of cars in the rental system is kept the same, the sum of car inventories at all stations and the number of cars in leasing should be kept at *S* at the start of period $t \geq 2$. At the start of period *t*, the number of cars at all stations is $\sum_{i=1}^{M} v_{i,t}$, and the number of cars leased is O_{t-1}. Thus, the following constraint holds.

$$\sum_{i=1}^{M} v_{ti} + O_{t-1} = S,\ \forall t \geq 2 \tag{13}$$

4. Solution Method

Lemma 1: For all t, i, k, and b_{tik} can be expressed as follows.

$$b_{tik} = \begin{cases} O_{i,K+t-1} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t} q_{xji,k+t-x}^{e},\ \forall i,k,t < K \\ \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=t-K+k}^{t} q_{xji,k+t-x}^{e},\ \forall i,k \geq K \end{cases} \tag{14}$$

Proof:

It is clear that b_{1ik} in Eq. (6) can be expressed by the first term of Eq. (14).

Next, the proof shows the statement that Eq. (7) can be expressed by Eq. (14).

For $t = 2$, Eqs. (6) and (7),

$$b_{2,ik} = O_{i,k+1} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{1ji,k+1}^{e} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{2jik}^{e} = O_{i,k+1} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{2} q_{xji,k+2-x}^{e}$$

Thus, the statement holds for $t = 2$ for Eq. (7).

By induction, substituting $b_{t-1,i,k+1} = O_{i,k+t-1} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1} q_{xji,k+t-x}^{e}$ into b_{tik} gives

$$b_{tik} = O_{i,k+t-1} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1} q_{xji,k+t-x}^{e} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjik}^{e} = O_{i,k+t-1} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t} q_{xji,k+t-x}^{e}$$

Thus, the proof for the first term of Eq. (14) is completed.

For $t = K$,

$$b_{K,ik} = b_{K-1,i,k+1} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{Kjik}^{e}$$

since $b_{K-1,i,k+1} = O_{i,k+K-2} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1} q_{xji,k+K-x}^{e}$

and $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1} q_{xji,k+K-x}^{e} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{Kjik}^{e} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K} q_{xji,k+K-x}^{e}$

Thus, the second term of Eq. (14) holds true for $t = k$ and $k < K$.

Again by induction, substituting $b_{t-1,i,k+1} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1} q_{xji,k+K-x}^{e}$ into b_{tik} gives

$$b_{tik} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-k+1} q_{t-x,ji,k+x}^{e} + \sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjik}^{e} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=t-K+k}^{t} q_{xji,k+t-x}^{e}$$

Finally, for $k = K$, from the second term of Eq. (14), the result of

$b_{tiK} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=t-K+K}^{t} q_{xji,K+t-x}^{e} = \sum_{e=1}^{E}\sum_{j=1}^{M} q_{tjiK}^{e}$ holds and, thus, completes the proof.

Lemma 2: The value of $O_t \forall, t, K > 1$ can be expressed as follows.

$$O_t = \begin{cases} \sum_{i=1}^{M}\sum_{k=1}^{K-t} O_{i,K+t} + \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{x=t}^{t}\sum_{k=t-x+2}^{K} q_{xjik}^{e},\ \forall t < K \\ \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{x=t-K+2}^{t}\sum_{k=t-x+2}^{K} q_{xjik}^{e},\ \forall i,k,t \geq K \end{cases} \tag{15}$$

Proof:

By Eqs. (10) and (14),

$$O_2 = \sum_{i=1}^{M}\sum_{k=2}^{K} b_{2ik} = \sum_{i=1}^{M}\sum_{k=2}^{K} O_{i,k+1} + \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=2}^{K}\sum_{x=1}^{t} q_{xji,k+t-x}^{e}$$

Note $\sum_{k=2}^{K} O_{i,k+1} = \sum_{k=1}^{K-2} O_{i,k+2}$ due to $O_{i,K+1} = 0$ and $\sum_{x=1}^{2}\sum_{k=2}^{K} q_{3-x,ji,k+x-1}^{e} = \sum_{x=1}^{2}\sum_{k=2-x+2}^{K} q_{xjik}^{e}$

Thus, the result holds for $t = 2$.

By Eqs. (10) and (14), for $t < K$,

$$O_t = \sum_{i=1}^{M}\sum_{k=2}^{K} b_{tik} = \sum_{i=1}^{M}\sum_{k=2}^{K} O_{i,k+t-1} + \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{x=1}^{t}\sum_{k=2}^{K} q_{xji,k+t-x}^{e}$$

Since $O_{i,k} = 0$ for $k > K$, $\sum_{k=2}^{K} O_{i,k+t-1} = \sum_{k=1}^{K-t} O_{i,k+t}$ and

since $\sum_{x=1}^{t}\sum_{k=2}^{K} q_{xji,k+t-x}^{e}$ can be rearranged as $\sum_{x=1}^{t}\sum_{k=t-x+2}^{K} q_{xjik}^{e}$,

the first term of Eq. (15) follows. For $t \geq K$, by Eqs. (10) and (14),

$$O_t = \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=2}^{K}\sum_{x=t-K+k}^{t}\sum_{k=2}^{K} q_{xji,k+t-x}^{e}$$

Since $q_{t-x+1,ji,k}^{e} = 0$ for $k > K$ and $\sum_{k=2}^{K}\sum_{x=t-K+k}^{t}\sum_{k=2}^{K} q_{xji,k+t-x}^{e}$ can be rearranged as $\sum_{x=t-K+2}^{t}\sum_{k=t-x+2}^{K} q_{xjik}^{e}$.

Thus, $O_t = \sum_{e=1}^{E}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{x=t-K+2}^{t}\sum_{k=t-x+2}^{K} q_{xjik}^{e}$ and the second term of Eq. (15) holds for $t \geq K$.

Lemma 3: For $t \leq K$ and all i and v_{ti} can be expressed as follows.

$$v_{ti} = \begin{cases} s_i + \sum_{x=1}^{t-1} O_{ix} + h_{ti}^1 - h_{ti}^2, \ \forall t < K \\ s_i + h_{ti}^1 - h_{ti}^2, \ \forall t \geq K \end{cases} \tag{16}$$

where $h_{1i}^1 = h_{1i}^2 = 0$,

$$h_{ti}^1 = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1}\sum_{k=1}^{t-x} q_{xjik}^{e}, \ \forall t > 1 \tag{17}$$

$$h_{ti}^2 = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1}\sum_{k=1}^{K} q_{xjik}^{e}, \ \forall t > 1 \tag{18}$$

Proof:

By Eq. (11), $v_{2,i} = v_{1i} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{1ijk}^{e} + b_{1i1}$.

By Eqs. (12) and (14), v_{2i} can be expressed as $v_{2i} = s_i - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{1ijk}^{e} + O_{i1} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{1} q_{xji,2-x}^{e}$

Note $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{1ijk}^{e}$ is equal to $u_{ti}^2$and $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{1} q_{xji,2-x}^{e}$ can be rewritten as

$$\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{1}\sum_{k=1}^{2-x} q_{xjik}^{e} = h_{ti}^1$$

Thus, the first term of Eq. (16) holds for $t = 2$. For $3 \leq t < K$, by Eq. (11),

$$v_{t+1,i} = v_{ti} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e} + b_{ti1}$$

By induction,

$$v_{t+1,i} = s_i + \sum_{x=1}^{t-1} O_{ix} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1}\sum_{k=1}^{t-x} q_{xji,k}^{e} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{x=1}^{t-1} q_{xijk}^{e} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e} + O_{it} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t} q_{xji,t+1-x}^{e}$$

in which since $\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t-1}\sum_{k=1}^{t-x} q_{xji,k}^{e} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t} q_{xji,t+1-x}^{e}$ can be rearranged as

$$\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{t}\sum_{k=1}^{t+1-x} q_{xji,k}^{e} = g_{t+1,i}^1$$

and $-\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{x=1}^{t-1} q_{xijk}^{e} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{tijk}^{e}$ can be rearranged as

$$-\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{x=1}^{t} q_{xijk}^{e} = -h_{ti}^2$$

Thus, the first term of Eq. (16) holds.

For $t = K$, $v_{Ki} = v_{K-1,i} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{K-1,ijk}^{e} + b_{K-1,i1}$.

By the first term of Eq. (16),

$$v_{K-1,i} = s_i + \sum_{x=1}^{K-2} O_{ix} + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-2}\sum_{k=1}^{K-1-x} q_{xji,k}^{e} - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-2}\sum_{k=1}^{K} q_{xijk}^{e}$$

and by Eq. (14), $b_{K-1,i1} = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1} q_{xji,K-x}^{e}$.

Thus, v_{Ki} can be rewritten as $v_{Ki} = s_i + \sum_{x=1}^{K-2} O_{ix} + B_{K-1}^1 + B_{K-1}^2$ where

$$B_{K-1}^1 = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-2}\sum_{k=1}^{K-1-x} q_{xji,k}^e + \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1} q_{xji,K-x}^e$$

and

$$B_{K-1}^2 = -\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K} q_{K-1,ijk}^e - \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-2}\sum_{k=1}^{K} q_{xijk}^e$$

Note B_{K-1}^1 can be rewritten as

$$B_{K-1}^1 = \sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1}\sum_{k=1}^{K-x} q_{xji,k}^e = u_{K,i}^1$$

and

$$B_{K-1}^2 = -\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{x=1}^{K-1}\sum_{k=1}^{K} q_{xijk}^e = u_{Ki}^2$$

Thus, the second term of Eq. (16) holds for $t = K$. For $t > K$, the proof of the second term of Eq. (16) is similar to the first term of Eq. (16) except that $O_{it} = 0$ for $t > K$ since cars still leased at the beginning of the first period must be returned within the maximum lease periods of K. Thus, the result follows.

Lemma 4: The equation of $\sum_{i=1}^{M} v_{ti} + O_{t-1} = S$ in Eq. (13) can be expressed by:

$$\sum_{i=1}^{M}\left(h_{ti}^1 - h_{ti}^2\right) = \begin{cases} \sum_{i=1}^{M}\sum_{k=t}^{K} O_{ik}, \ \forall t < K \\ \sum_{i=1}^{M}\sum_{k=t}^{K} O_{ik}, \ \forall t \geq K \end{cases} \tag{19}$$

Proof:

For $t < K$, from Eq. (16),

$$\sum_{i=1}^{M} v_{ti} + O_{t-1} = \sum_{i=1}^{M} s_i + \sum_{i=1}^{M}\sum_{k=1}^{t-1} O_{ik} + \sum_{i=1}^{M}(h_{t,i}^1 - h_{t,i}^2)$$

Note $\sum_{i=1}^{M} s_i + \sum_{i=1}^{M}\sum_{k=1}^{t-1} O_{ik} = S - \sum_{i=1}^{M}\sum_{k=t}^{K} O_{ik}$.

Thus, $\sum_{i=1}^{M} v_{ti} + O_{t-1} = S$ can be rearranged as $\sum_{i=1}^{M}(h_{t,i}^1 - h_{t,i}^2) = \sum_{i=1}^{M}\sum_{k=t}^{K} O_{ik}$ for $t < K$.

For $t \geq K$, from Eq. (16),

$$\sum_{i=1}^{M} v_{ti} + O_{t-1} = \sum_{i=1}^{M} s_i + \sum_{i=1}^{M}(h_{t,i}^1 - h_{t,i}^2)$$

Note $\sum_{i=1}^{M} s_i = S - \sum_{i=1}^{M}\sum_{k=1}^{K} O_{ik}$.

Thus, $\sum_{i=1}^{M} v_{ti} + O_{t-1} = S$ can be rearranged as $\sum_{i=1}^{M}(g_{t,i}^1 - h_{t,i}^2) = \sum_{i=1}^{M}\sum_{k=1}^{K} O_{ik}$ for $t \geq K$.

Lemma 5: constraint Eq. (2) can be expressed as follows.

$$\sum_{e=1}^{E} w_{ti}^e = \begin{cases} s_i + \sum_{x=1}^{t-1} O_{ix} + h_{ti}^1 - h_{ti}^2, \ \forall t < K \\ s_i + h_{ti}^1 - h_{ti}^2, \ \forall t \geq K \end{cases} \tag{20}$$

Proof: It is clear from *Lemma 3*.

According to *Lemmas 1* to *4*, Eq. (2) can be replaced with Eq. (20) by *Lemma 5*, Eqs. (6)-(8) can be replaced with Eq. (14) by *Lemma 1*, Eqs. (9)-(10) can be replaced with Eq. (15) by *Lemma 2*, Eqs. (11)-(12) can be replaced with Eq. (16) by *Lemma 3*, and Eq. (13) can be replaced with Eq. (19) by *Lemma 4*. Thus, the original model can be rewritten as follows:

Model 2: Max PR subject to constraints of Eqs. (3)-(5), (14), (16), and (19)-(20).

This article refers to the above model as Model 2. The number of cars in leasing after car return at the end of the period *t* of O_t can be computed by Eq. (15) after solving Model 2. In Model 2, the decision variables only contain variables q_{tijk}^e, u_{ti}^e, and w_{ti}^e. In addition, since the model is linear, it can be quickly and optimally solved by linear programming solvers.

4.1. Dynamic rental car allocation

One can extend the above model to the case with the ability to deal with dynamic demand. When applying the proposed model to the problem with random demand, the ending inventory of each rental station and the status of the cars leased out differs from the forecast status at the beginning of the planning period. After the experiment of all the subperiods of a period, the solution procedure can recalculate Model 2 to revise the allocation strategy for all subsequent periods to match the actual inventory and lease statuses. Fig. 1 shows how to use the proposed linear programming model to generate dynamic decisions. Suppose the decision maker wants to recalculate the allocation and rental strategy in period t.

In this situation, the solution procedure will first modify the following parameters: the number of planning periods, T, the number of available cars at rental station i at the beginning of the recalculate period, s_i, and the number of leased cars customers will return to station i at the end of period $t+k-1$, O_{ik}. The three parameters are named initial state parameters. The renewal process of them is as follows:

(a) The remaining planning periods of T is updated to T-t.

(b) The current car level at station i is set to $s_i = v_{ti}$.

(c) The number of cars in leasing customers will return $k-1$ periods later is set to $O_{ik} = b_{tik}$.

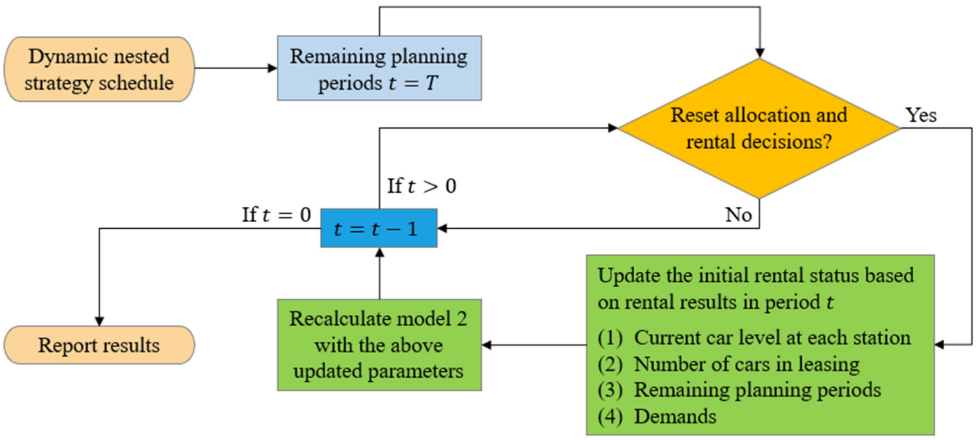

Fig. 1 Framework of dynamic resource allocation

4.2. Structure of the exclusive allocation approach

The objective function of the exclusive allocation model is the same as Eq. (1) in the nested allocation model. The constraints or equation constraints for acceptable rental cars, the number of cars in leasing, inventory level at rental stations, and car inventory conservation are the same as those in Eq. (2) and Eqs. (5)-(13) in the nested allocation model, except that the constraints of Eqs. (3)-(4) for nested structure and booking limit in the nested allocation model should be modified. Under the exclusive allocation approach, w_{it}^e is denoted as the rental threshold for channel e at rental station i in period t. Different from the nested allocation approach, the values of w_{it}^e and v_{it} have a relationship of $\sum_{e=1}^{E} w_{it}^e = v_{it}$. However, under the exclusive allocation approach, customers' requests are accepted only if $q_{i\tau}^e < w_{it}^e$ where τ is a sub-period within a period, ranging from 1 to N. Thus, constraints Eqs. (3)-(4) in the nested structure model are replaced with constraint Eq. (21).

$$\sum_{j=1}^{M} \sum_{k=1}^{K} q_{tjik}^e \le w_{ti}^e, \ \forall e,i,t \tag{21}$$

5. Design Examples

This section designs the following examples to illustrate the performance of the proposed model. Suppose that the parameters used in the numerical examples are related to the leasing station, leasing pipeline, leasing return station, and the number of phases. The rental fee for channel e is 130 + 20e dollars per period, with a discount of 3 dollars for each additional rental period. In addition, customers shall pay two times the difference between the rental station i and return station j. That is, for rental type (t,i,j,k) at channel e, the rental revenue is set at $r_{tijk}^e = (130 + 20 \times e)k - 3(k-1) + 2|i-j|$ dollars for $|i-j| \le 4$. The shortage cost per car for all periods t is set at \$10k. The demand probability function p_{ijktd}^e that d requests for rental type (t, i, j, k) at channel e is generated by $p^{e}_{ijktd} = P_{ti}P_{eijkd}$, where P_{ti} is the probability that requests are present at station i in any subperiod of period t, and P_{eijkd} is the probability that d customers will rent cars at station i for k periods in sales channel e and return their cars at rental station j.

Appropriate prediction models can predict the precise demand probability function. However, this topic is out of the scope of this study. This article assumes that the value of P_{ti} changes cyclically every three periods and every five rental stations, respectively. The value is supposed to be P_{ti} = 0.75 − 0.03 mod[t,3] + 0.02 mod[i,5], where the symbol mod[b, g] is the remainder of b divided by g. For the generation of P_{eijkd}, this article first gives each combination (e, i, j, k, d) a weight x_{eijkd} and then generates it through $P_{eijkd} = x_{eijkd}/\sum_{e=1}^{E}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{d=1}^{D}x_{eijkd}$. In the previous paragraph, this article has set that the rental fee for the rental channel e' is higher than channel e for all $e' > e$, and the distance between rental station i and return station j increases as the difference between i and j increases. Here, the weight x_{eijkd} is generated based on the following logic. The weight decreases with increased distance between rental station i and return station j, the channel's fare, and the rental duration k. The formula to generate x_{eijkd} is x_{eijkd} = 80 − 10e − H(i,j) − k − 10d, where H(i,j) = 2 mod(|i − j|,3) for |i − j| ≤ 3 and H(i,j) = 6 for |i − j| > 3.

The solution procedure accumulates the probability of each possible requesting rental type to create a roulette distribution probability chart for simulating whether rental requests are present on each lease station in each subperiod of a certain period and confirming the type and quantity when rental requests occur. In the simulation stage, the solution procedure generates a random number for each subperiod for each rental station according to the position where the random value falls in the roulette to determine the rental type and quantity of the appearing requests. Below, the solution procedure simulates a small-scale example and two additional large-scale examples.

Example 1: *An illustrative example*

This example is mainly used to illustrate the application of the proposed model and how the proposed nested approach brings higher profit than the traditional resource allocation method. Below, the abbreviations NA and EAA represent the nested allocation method and the exclusive allocation approaches, respectively. This example sets the parameters of planning periods T, number of rental stations M, number of rental channels E, maximum rental periods K, and number of subperiods in a period N to small values at T = 2, N = 6, M = 3, E = 2, and K = 1, respectively, to illustrate the booking process in detail. The number of cars is set at s_i = 5 cars for all rental stations. Table 2 shows this example's booking limits for NA and EAA approaches.

Table 2 Booking limits for nested allocation u^{e}_{it} and exclusive allocation w^{e}_{it}

e	u^{e}_{it}			w^{e}_{it}		
	i = 1	i = 2	i = 3	i = 1	i = 2	i = 3
1	6	5	4	3	9	2
2	6	6	6	4	10	4

In the following tables, v^{n}_{i}, g^{en}_{i}, and q^{en}_{i} are used to represent the car inventory at the beginning of the nth subperiod at station i, the cumulative number of cars leased over channels 1 + e until the beginning of the nth subperiod at station i, and the number of cars rented out during this period, respectively. In subperiod n, acceptance of new demand depends on whether the value of g^{en}_{i} is less than u^{e}_{i1} and whether cars are still in stock. The value of v^{n}_{i} is equal to the inventory of the previous subperiod, that is, v^{n-1}_{i}, minus the rental cars at the nth subperiod, that is, $\sum_{e'=1}^{E} q^{e'}_{i,n-1}$. Thus, it is updated by the formula below.

$$v_i^n = v_i^{n-1} - \sum_{e'=1}^{E} q_{i,n-1}^{e'}, \ \forall i, n > 1 \tag{22}$$

Suppose q_{in}^{e} rental demands at channel e in subperiod n are accepted. At the start of subperiod n + 1, the cumulated accepted cars of $g_i^{e,n+1}$ for channels 1 to e, $g_i^{e+1,n+1}$ for channels 1 to e + 1, and $g_i^{E,n+1}$ for channels 1 to E at the end of subperiod n should be updated by adding q_{in}^{e}. Thus, $g_i^{e',n+1}$ for $e' \geq e$ is updated for all $e' \geq e$ channels with equation $g_i^{e',n+1} = g_i^{e',n} + q_{in}^{e}$for all $e' \geq e$ channels with Eq. (23).

$$g_i^{e',n+1} = g_i^{e'n} + q_{in}^{e},\ \forall i, n < N, e' \geq e \tag{23}$$

The rental demand for each station for each subperiod is generated based on the falling points of random numbers on the previously designed roulette distribution probability chart. Given that each period's lease operation process is similar, this paper only illustrates the first period's lease process to save space. The rental process for other periods is similar to that of the first period. Table 3 shows the randomly generated demand for all subperiods of period 1. This table shows that, in subperiod n = 1, d = 1 requests are present for rental type $(t, i, j, k) = (1,1,1,1)$ at channel 2 at rental station 1, d = 1 requests are present for rental type $(t, i, j, k) = (1,2,3,1)$ at channel 2 at rental station 2, and d = 3 requests are present for rental type $(t, i, j, k) = (1,3,2,1)$ at channel 1 at rental station 3.

Table 3 Appearing requests in each subperiod of period 1

n	1			2			3			4			5			6		
i	e	j	d	e	j	d	e	j	d	e	j	d	e	j	d	e	j	d
1	2	1	1	0	0	0	0	0	0	1	2	1	1	3	2	1	1	2
2	2	3	1	2	2	3	0	0	0	1	2	1	1	1	3	2	2	1
3	1	2	3	0	0	0	2	2	1	1	2	3	2	3	1	1	3	3

The number of 0 in column e represents that no demand occurs.

Table 4 shows that, at the start of subperiod n = 1, the nested cumulative leased cars are g_i^{en} = 0 for all e and i. The acceptable rental request for station 1 at channel 2 is $u_{it}^{e} - g_i^{en} = u_{11}^{2} - g_1^{21} = 6$, that for station 2 at channel 2 is $u_{21}^{2} - g_2^{21} =$ 6, and that for station 3 at channel 1 is $u_{31}^{1} - g_3^{11} = 4$. Thus, the requests for d = 1 unit of rental type $(1,1,1,1)$ at channel 2 at rental station 1, d = 1 unit of rental type $(1,2,3,1)$ at channel 2 at rental station 2, and d = 3 units of rental type $(1,3,2,1)$ at channel 1 at rental station 3 shown in Table 3 are all accepted.

Using Eq. (14), at the start of subperiod 2, the values of $(g_i^{1,2}, g_i^{2,2})$ at stations 1, 2, and 3 are (0,1), (0,1), and (3,3), respectively (Table 4). In subperiod 2, 3 units of rental type $(1,2,2,1)$ at channel 2 are accepted because the sum of $g_2^{2,2}$ plus the newly accepted 3 rentals are equal to 4 and less than $u_{it}^{e} = u_{21}^{2} = 6$. Rental logic is applied to each of the following subperiods, and all the results are shown in Table 4. The situation in the fourth period is specifically explained. In subperiod 4, the acceptable rentals for station 3 at channel 1 are $u_{it}^{e} - g_i^{en} = u_{31}^{1} - g_3^{14} = 4 - 3 = 1$. Thus, although 3 rental requests are present for the rental type $(1,3,2,1)$ at channel 1, the system only can accept 1 unit.

Table 4 Booking process for period 1 under nested approach

n			1			2			3			4			5			6		
i	e	u_i^e	v_i^r	g_i^e	q_i^e	v_i^r	g_i^e	q_i^e	v_i^r	g_i^e	q_i^e	v_i^r	g_i^e	q_i^e	v_i^r	g_i^e	q_i^e	v_i^r	g_i^e	q_i^e
1	1	5	6	0	0	5	0	0	5	0	0	5	0	1	4	1	2	2	3	2
	2	6		0	1		1	0		1	0		1	0		2	0		4	0
2	1	5	6	0	0	5	0	0	2	0	0	2	0	1	1	1	1	0	2	0
	2	6		0	1		1	3		4	0		4	0		5	0		6	0
3	1	4	6	0	3	3	3	0	3	3	0	2	3	1	1	4	0	0	4	0
	2	6		0	0		3	0		3	1		4	0		5	1		6	0

Table 5 shows the booking process for period 1 under the EAA approach. Under this approach, the booking limits of $(w_{i1}^{1}, w_{i1}^{2}, y_i^{2,2})$ at stations i = 1, i = 2, and i = 3 are (3,3), (2,4), and (5,6), respectively. Rental requests at subperiod 1 are all accepted because the rental requests, q_i^{en}, do not exceed their booking limits w_{it}^{e}. At the start of subperiod 2, the values of

$(y_i^{1,2}, y_i^{2,2})$ at stations i = 1, i = 2, and i = 3 are (0,1), (0,1), and (3,0), respectively, where y_i^{en} is the total number of accepted rentals from subperiod 1 to subperiod n = 1. In subperiod n = 2, d = 3 requests are present for rental type $(1,2,2,1)$ at channel 2 for station 2. However, under the exclusive allocation approach, the resources of each channel cannot be mutually supported. Thus, one of the three requests is rejected because only $w_{it}^e - y_i^{en} = w_{2t}^2 - y_2^{22}$ = 3 – 1 = 2 rentals can be accepted. The same logic is applied in the following subperiods.

Table 5 Booking process for period 1 under exclusive allocation approach

n			1			2			3			4			5			6		
i	e	w_{i1}^e	v_i^n	y_i^{en}	q_i^{en}	v_i^n	y_i^{en}	q_i^{en}	v_i^n	y_i^{en}	q_i^{en}	v_i^n	y_i^{en}	q_i^{en}	v_i^n	y_i^{en}	q_i^{en}	v_i^n	y_i^{en}	q_i^{en}
1	1	3	6	0	0	5	0	0	5	0	0	5	0	1	4	1	2	2	3	0
	2	3		0	1		1	0		1	0		1	0		1	0		1	0
2	1	2	6	0	0	5	0	0	3	0	0	3	0	1	1	1	2	0	3	0
	2	4		0	1		1	2		3	0		3	0		3	0		3	0
3	1	5	6	0	3	3	3	0	3	3	0	2	3	0	1	3	0	1	3	0
	2	6		0	0		0	0		0	1		1	0		1	1		2	0

The details of the accepted rentals under the nested and exclusive allocation approaches are summarized in Tables 6 and 7, respectively. The revenue obtained from these results is as follows. The total revenue for the NA approach is 5,466, while the revenues for channels 1 and 2 are 2,952 and 2,514, respectively. The total revenue for the exclusive allocation approach is 4,634, while the revenues for channels 1 and 2 are 2,310 and 2,324, respectively. It's worth noting that the profit of the nested approach is 17.9% higher than that of the exclusive allocation approach, reaffirming the success of the chosen strategy.

Table 6 Accepted rentals from the nested approach

t	1									2								
i	i = 1			i = 2			i = 3			i = 1			i = 2			i = 3		
j	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
e = 1	2	1	2	1	1	0	0	4	3	3	0	0	0	5	0	2	1	0
e = 2	1	0	0	0	3	1	0	1	1	1	0	0	4	0	0	3	0	0

Table 7 Accepted rentals from the exclusive allocation approach

t	1									2								
i	i = 1			i = 2			i = 3			i = 1			i = 2			i = 3		
j	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
e = 1	0	1	2	2	1	0	0	3	0	2	0	0	4	0	0	0	1	0
e = 2	1	0	0	0	2	1	0	1	1	1	0	0	4	0	0	3	0	0

Examples 2-4: *Results and discussion*

Through the three examples, this study intends to understand whether the NA approach is still better than the EAA approach for large-size rental systems. The parameters of planning periods T, number of rental stations M, maximum rental periods K, and number of subperiods in a period N are set at T = 7, N = 15, E = 3, and K = 2 for all examples. At the start of period 1, 15 cars are available for each rental station for all examples. The parameters of the number of rental stations M are set at 10, 20, and 50 for *Examples 2*, *3*, and *4*, respectively. Each example is simulated with 50 runs. The total profit by the EAA and the NA from the 50 simulations of *Examples 2*, *3*, and *4* are shown in Figs. 2, 3, and 4, respectively.

In these figures, the y-axis represents the income and the x-axis represents the number of the experiment case. The profits for the EAA approach fluctuate among the range of \$63,423 to \$82,100, \$135,694 to \$165,264, and \$293,809 to \$323,090, for *Examples 2*, *3*, and *4*, respectively. The profits for the NA approach fluctuate among the range of \$99,240 to \$107,514, \$215,370 to \$227,740, and \$568,606 to \$585,508, for *Examples 2*, *3*, and *4*, respectively. These results show that the NA approach outperforms the EAA approach, with significantly higher total profit. Tables 8, 9, and 10 summarize the results and show more detailed information related to the average profit (AP) per channel, total AP, minimum profit found, maximum profit found, and standard deviation (SD) of profit of the two examples.

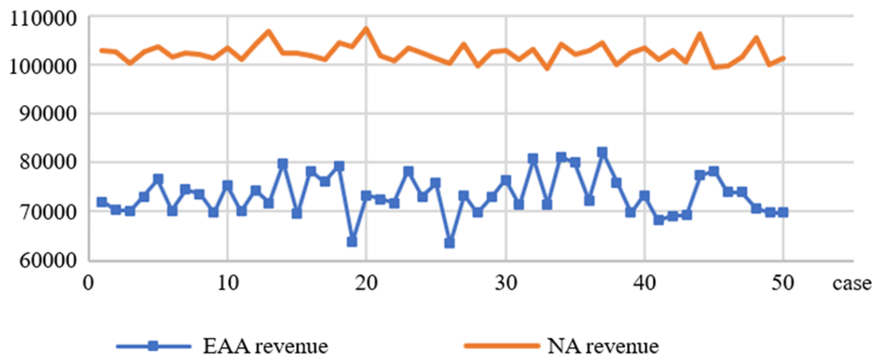

Fig. 2 Framework of dynamic resource allocation

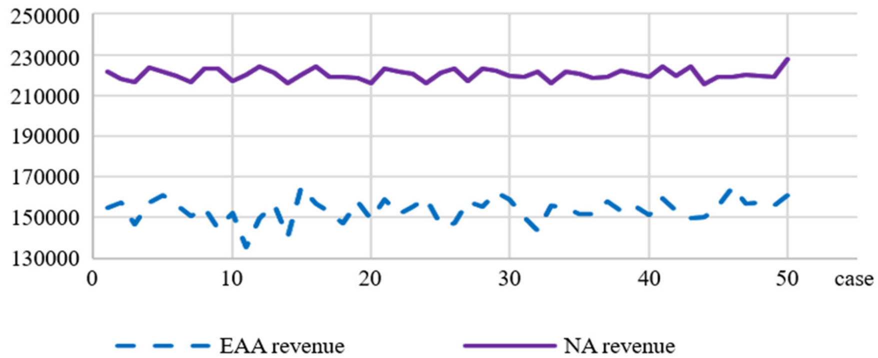

Fig. 3 Fifty simulation results of Example 3

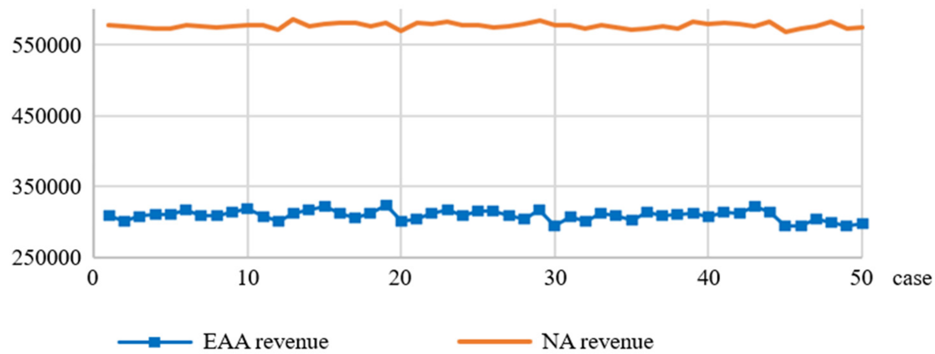

Fig. 4 Fifty simulation results of Example 4

In terms of the total AP, Tables 8, 9, and 10 show that the profits of the NA approach are 39.64%, 43.17%, and 86.9% higher than those of the EAA approach for Examples 2, 3, and 4, respectively. Regarding the minimum and maximum profits found by the three approaches, the lowest and highest profits by the NA approach are all higher than the EAA approach for all examples. In terms of the profit SD, the NA approach achieves profit SD values of approximately 44.19%, 46.08%, and 51.96% of those of the EAA approach. Regarding the average computation time per simulation (AT), Tables 7-9 show that the calculation times of the NA and EAA methods are very close, and the calculation times of both methods will not increase significantly as the number of rental stations increases.

Table 8 Computational result of Example 2

Profit	Channel			AP	Min	Max	SD	AT (seconds)
	1	2	3					
Nested	$44,220	$29,461	$28,775	$102,456	$99,240	$107,514	$1,818	202
Exclusive	$2,303	$36,195	$34,874	$73,372.8	$63,423	$82,100	$4,113	198

Table 9 Computational result of Example 3

Profit	Channel			AP	Min	Max	SD	Time (seconds)
	1	2	3					
Nested	\$99,547	\$61,238	\$59,431	\$220,216	\$215,370	\$227,740	\$2,672	402
Exclusive	\$12,473	\$74,882	\$66,461	\$153,816	\$135,694	\$165,264	\$5,799	391

Table 10 Computational result of Example 4

Profit	Channel			AP	Min	Max	SD	Time (seconds)
	1	2	3					
Nested	\$271,611	\$161,047	\$144,446	\$577,103.4	\$568,606	\$585,508	\$3,816.4	1,118
Exclusive	\$75,204	\$129,122	\$104,459	\$308,784.5	\$293,809	\$323,090	\$7,344.2	1,095

6. Conclusions

This study addressed a multichannel car rental system and developed a period-by-period linear programming model to construct a dynamic nested strategy of multiple rental stations with various sales channels. The established dynamic nested structure can allow managers to adequately adjust the number of rental cars in each rental channel, thereby increasing revenue. The main conclusions of this paper are as follows.

(1) The computational results show that the proposed nested approach can enhance revenue for multichannel car rental systems while maintaining proper storage space and short computation time.

(2) Regarding the total AP and SD values, the nested approach performs better than the exclusive allocation approach regarding solution stability and profit pursuit ability.

(3) When applying the proposed model to the situation with random demand, the ending inventory of each rental station and the status of the cars leased out differ from the forecast status at the beginning of the planning period. After the experiment of each period's subperiods, the solution procedure can recalculate the allocation strategy for subsequent periods to deal with the cases with random demands and match the actual inventory and lease statuses.

7. Limitation and Future Direction

Generally, a car rental system will provide a variety of car models. When customers cannot rent a car model they originally intended to rent, they may switch to leasing other car models. However, the framework of this article is limited to the rental problem of a single-car model and, therefore, does not consider the substitutability of demand between different car models. Ignoring substitution demand may lead to underestimating the benefits of the substitution effect among multi-car models. Thus, it is necessary to establish mathematical models and decision-making plans with alternative demands among different car models to improve the overall rental revenues of all car models.

However, the model of multiple rental car models will be a more complex problem based on a nested decision-making structure. In terms of development, it will also face more challenges. This study leaves this issue for future research. Additionally, this study did not provide in-depth estimates of online rental demand for each channel or explore parameters such as car rental and return. Given that these parameters significantly influence the quality of decisions, this study also identifies them as critical areas for future research with the potential to have a significant impact on operations management.

Conflicts of Interest

The authors declare no conflict of interest.

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