

Advantages of Multi GNSS Constellation: GDOP Analysis for GPS, GLONASS and Galileo Combinations

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Abstract

Positioning techniques made lots of progress in the last decades, thanks to the wide usage of the Global Navigation Satellite Systems (GNSS). During a satellite survey, interruption or complete absence of positioning service can happen due to obstacle presence or constrained environments. To avoid these problems, it is suitable to simulate a positioning survey determining the number of the GNSS satellites in view and their availability trend for a selected location. Using more than one constellation the number of the observed satellites is increased and the continuity and reliability of positioning significantly improved. The aim of this paper is to assess the impact of multi-GNSS constellation on positioning calculation in terms of number of available satellites and geometrical distribution in the sky. A simulation is conducted for different cut-off angles, ranging from 0° to 30° : satellites visibility predictions are performed for the city of Benevento (Italy) using short observing sessions (96 daily) and considering GPS, GLONASS and GALILEO constellation. The benefits of their combinations are investigated: in order to assess the observation quality, the Geometrical Dilution of Precision (GDOP) is used as criteria to prove how it is possible to reduce degradation of the position accuracy by using multi-GNSS combinations. Particularly, GPS+GLONASS supplies higher performances compared to the other solutions. Because the low number of satellites in view, the contribution of GALILEO is limited, and its presence instead of GPS or GLONASS in the two constellation solutions produces a decrease in positioning accuracy.

Keywords: Multi-constellation, satellite geometry, GNSS data quality, GDOP

1. Introduction

Global Navigation Satellite System (GNSS) is an indispensable tool for all professionals working in the delivered geodetic and topographic positioning [1], predominantly through a faster and better quality of results compared to other surveying techniques [2]. It permits to locate the user, through calculations involving information from a number of satellites. Using the timing and positioning data encoded in the signals transmitted by each satellite, GNSS receiver on or near the earth's surface determines its location. Processing each signal the receiver calculates the distance (from the transmission time delay) between it and the transmitting satellite. If accurately determined, three measurements can be used to locate a point [3]. Unfortunately, the clock in GPS receiver is not as accurate as the very precise and expensive atomic clock in each satellite. Hence, three measurements do not permit to determine exactly the position coordinates. In fact, if the two clocks are off by only a small fraction, significant errors can be introduced in the position data calculation. To determine the unknown three position coordinates and one clock error, four satellites at least are necessary. In other terms, managing current signal data from four or more satellites enables the receiver to determine its position.

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According to Gebre-Egziabher and Gleason [4], it is possible to group GNSS applications into at least nine different categories: Personal navigation; Aviation applications; Weak signal navigation; Automotive applications; Agriculture, forestry, and natural resource exploration; Marine applications; Geodesy and surveying; Space applications; and Scientific applications. Because many of them require a high level of performance in terms of accuracy and robustness, to use multi GNSS constellations instead of the single one can be appropriate. A multi GNSS receiver able to calculate position, velocity and time by receiving the satellite signals broadcasted from multiple navigation satellite systems brings several improvements to high precision and real time applications. In literature several authors focus on the potentiality of GNSS integration [5-8]. Benefits have been widely documented: they consist of availability of additional observations and better satellite geometry, especially in obstructed environments. In an urban environment, the buildings and other obstacles can block or reflect satellites signals. It implies a reduced number of satellites in view and poor position accuracy. For this reason, a multi constellation receiver is required to collect the necessary signals number to calculate a trustworthy position solution.

In addition to the Global Positioning System (GPS), which maintained by the United States of America, [9] and the Global Navigation Satellite System (GLONASS), which maintained by Russia [10], the emerging satellite navigation systems maintained by China, that is BeiDou Navigation Satellite System, [11] and GALILEO, which maintained by the European Union, [12] provide the potential for more accurate and reliable GNSS applications [13]. The fusion of multiple GNSS can significantly increase the number of observed satellites, optimize the spatial geometry and improve accuracy, continuity and reliability of positioning [14]. The high cut-off elevation capability of multi-GNSS will significantly increase its applicability in constrained environments, such as in urban canyons and open pits [15].

The multi-GNSS constellations provide a better global coverage with multiple frequency observations: more satellites in view mean improved satellite geometry [16]. It is common practice to use Dilution of Precision (DOP) factors to find the best subset of satellites at any time: they describe the effect of the satellite-receiver geometry on the accuracy of point positioning [17]. There are different types of DOP, according to parameters that are used to estimate the quality of the satellite distribution in the sky. One of them is Geometrical Dilution of Precision (GDOP): it is a powerful and widespread quantity for determining the errors resulting from satellite configuration geometry [18].

The aim of this paper is to compare GDOP values considering different combinations of multi-GNSS constellation; particularly satellites visibility predictions are performed for a station located in the city of Benevento (Italy) using short observing sessions (96 daily) of GPS, GLONASS and GALILEO, introducing different values of cut-off angle (which is the angle below satellites should not be tracked), ranging from 0° to 30° .

The paper is organized as follows. The section 2 briefly expresses the concept of GDOP and its calculation for an integrated GNSS. Section 3 illustrates the related work for GNSS accuracy evaluation based on GDOP calculation: results obtained using each stand-alone constellation (GPS, GLONASS, GALILEO) as well as their combinations (GPS+GLONASS, GPS+GALILEO, GLONASS+GALILEO, GPS+ GLONASS+GALILEO) are shown and discussed. The section 4 describes the conclusion of the paper.

2. GDOP calculation for multi-GNSS constellations

GDOP today is a dimensionless single number that permits to evaluate the quality of satellite geometry for a given location and time [19]. Considering the square pyramid formed by lines joining four satellites with the receiver, the smaller the volume of the pyramid, the worse (higher) the value of GDOP, the larger its volume, the better (lower) the value of GDOP will be. Similarly, the greater the number of satellite, the better the value of GDOP will be. If the satellites are near into the sky then the geometry is weak considerable and received GDOP values will be high but if the satellites are far from each other geometry is strong considerable and received GDOP values will be low [20]. As the satellites move along their orbits, the GDOP changes

with time. GDOP measures the effectiveness of potential measurements to specify the precision and accuracy of the data received from GNSS satellites [21]. In other terms, GDOP describes the effect of geometry on the relationship between measurement error and position determination error [22]. Poor satellite geometry causes a temporal incapability to acquire positioning data (Fig. 1).

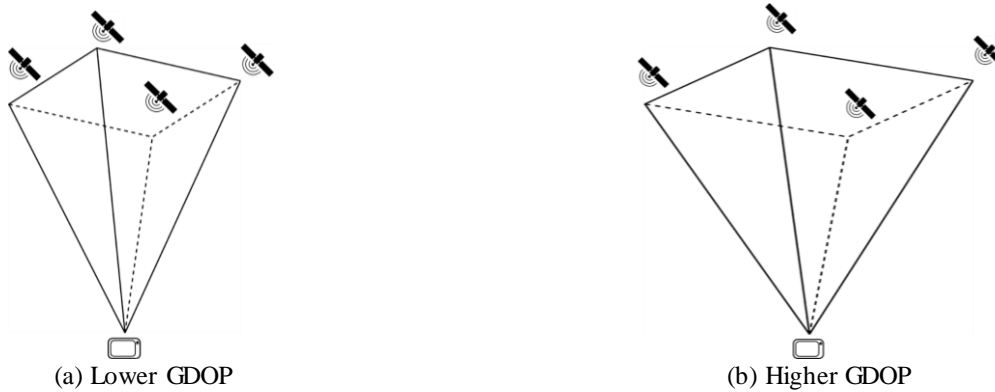


Fig. 1 Pyramids show respectively a lower and higher GDOP coefficient values which singly correspond to a good and bad satellites distribution

In a multi-GNSS context, the errors due to different coordinate and time systems between constellations have to be considered. The coordinate system errors are studied by Guo et al [23] establishing that for navigation applications this difference does not affect GDOP calculation. For what concerns the second kind of errors, Hofmann-Wellenhof, Lichtenegger and Walse [24] observe that the conventional GDOP, mainly investigated for single GNSS, is not valid for multi-GNSS. The time offsets can be determined at system level or at user level: in the first case, a receiver extracts the system time offsets among GNSSs from navigation message; at user level, the system time offset is calculated by the receiver using at least one additional satellite from each additional time reference frame (i.e. 5 satellites for GPS + GALILEO).

According to Teng and Wang [25], the GDOP for multi-GNSS constellations can be written as:

$$GDOP = \sqrt{tr[(H_n^T Q_n H_n)^{-1}]} \tag{1}$$

where

- H_n is the geometric matrix;

$$H_n = \begin{bmatrix} H_A & 1_A & 0_A & 0_A & \dots \\ H_B & 0_B & 1_B & 0_B & \dots \\ H_C & 0_C & 0_C & 1_C & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}^T \tag{2}$$

- Q_n is the weight matrix related to measurement noise with the multi-GNSS constellations.

$$Q_n = \begin{bmatrix} Q_A & & & \\ & Q_B & & \\ & & Q_C & \\ & & & \dots \end{bmatrix}^T \tag{3}$$

The subscripts $\gamma = A, B, C \dots$ indicate the different constellations. Assuming n_γ the number of tracked satellites in γ , $n = n_a + n_b + n_c + \dots$ represent the total satellites number.

The presence of ones and zeros vectors in different columns of H is motivated by the purpose to get each receiver clock bias, being it different for each constellation. The matrix Q_n is used to calculate GDOP for the multi-GNSS constellations: it requires appropriate weighting of single satellites range measurement. Because h_i represents the direction cosine vector between the receiver and the corresponding satellite, the matrix H_n is defined by:

$$H_A = \begin{bmatrix} h_1 \\ \dots \\ h_{n_A} \end{bmatrix}, H_B = \begin{bmatrix} h_{n_A+1} \\ \dots \\ h_{n_A+n_B} \end{bmatrix}, H_C = \begin{bmatrix} h_{n_A+n_B+1} \\ \dots \\ h_{n_A+n_B+1} \end{bmatrix}, \dots \tag{4}$$

Both the approximate receiver position and the corresponding satellite position allow determining the direction cosine vector h_i in the eq. (4).

Many researchers have focused their attention to find new methods to compute GDOP and thus select the best satellites subset. Du, Han, Lu, Wu and Zhang [26] obtain the expression of the mathematical expectation of GDOP using random variables azimuth and elevation, instead of the used three direction cosines. Once considered the different block situation of user, they import the minimal elevation angle as a constraint condition determining the expression of GDOP with the minimal elevation angle limitation.

Jwo and Lai [27] propose a neural network approach for navigation solution processing: it allows selecting the best satellites subset. Approximating or classifying the GDOP factors, it is possible to evaluate all subsets of satellites reducing the workload. Both the performance and the computational cost on neural network based GDOP approximation and classification are investigated: for what concern the accuracy, the used approaches are able to provide good performance, given enough time or enough training data.

Dong, Fu, Tian and Yang [28] study the relationship between GDOP and tetrahedral volume value to reduce the calculation load and improve the effect of satellite selection. For this reason, they introduce a new algorithm able to get the minimal GDOP value with a small computation amount. Simulation tests confirm the proposed algorithm validity.

Kang, Song and Xue [29] propose an adaptable and flexible method to select a navigation satellite subset based on a genetic algorithm. Their approach is aimed to minimize the factors in the GDOP. They use a modified genetic algorithm with an elite conservation strategy, adaptive selection, adaptive mutation and a hybrid genetic algorithm that can select a subset of the satellites maintaining position accuracy.

Doong [30], using Newton's identities from the theory of symmetric polynomials, develops an efficient closed-form formula to calculate GDOP giving as inputs the traces of the measurement matrix, its second and third powers and the determinant of the measurement matrix M . The following formula is adopted:

$$GDOP = \sqrt{\frac{0.5 h_1^3 - 15.5 h_1 h_2 + h_3}{3h_4}} \quad (5)$$

where h_1 , h_2 , h_3 are respectively the first, second and third degree power sum of the eigenvalues of M . It allows finding the decision boundaries of the defined classes directly from set of features instead of computing the GDOP.

Li and Zhu [31] propose a new satellite-selecting algorithm with the advantages to be simple, practical and easy to implement. It selects the optimal satellites for navigation by making full use of available information improving real-time performance and choosing better satellite combination.

3. Application

The goal of this paper is to demonstrate how it is possible to increase the satellites availability and reliability integrating several GNSS observations i.e. in urban areas where obstacles such as buildings can lead to a disruption of positioning service.

Currently, several soft wares perform satellite visibility prediction: one of them is Trimble Planning version 2.90 [32]. To simulate satellite visibility from a specified location for a given test date, session duration, intervals and cut-off angle, Trimble requires the latest Ephemeris file. It contains the basic orbital parameters for all considered satellites constellations to evaluate GDOP values (Table 1).

Table 1 Structure of an almanac file

Id	The Pseudo Random Noise (PRN) number of the satellite constellation
Health	Satellite health value
Eccentricity	The amount of the orbit deviation from circular (orbit)
Time of Applicability	The almanac reference time in seconds
Orbital Inclination	The angle to which the space vehicle orbit meets the equator
Rate of Right Ascension	Rate of change in the measurement of the angle of right ascension
SQRT(A) (m ^{1/2})	Square root of semi major axis
Right Ascen at Week	Geographic longitude of the ascending node of the orbital plane at the weekly epoch
Argument of Perigee	Angular measurement along the orbital path measured from the ascending node to the point of perigee
Mean Anom	Angle traveled past the argument of perigee
Af(0)	Clock bias in seconds
Af(1)	Space Vehicle clock Drift in seconds per seconds
Week	The almanac reference week

Simulation tests were performed on a station located in Benevento (Italy) which WGS84 coordinates are: $\phi = 41^{\circ} 07' 17.288''$ and $\lambda = 14^{\circ} 46' 40.751''$ whereas its height is 136 meters above the mean sea level (Fig. 2).

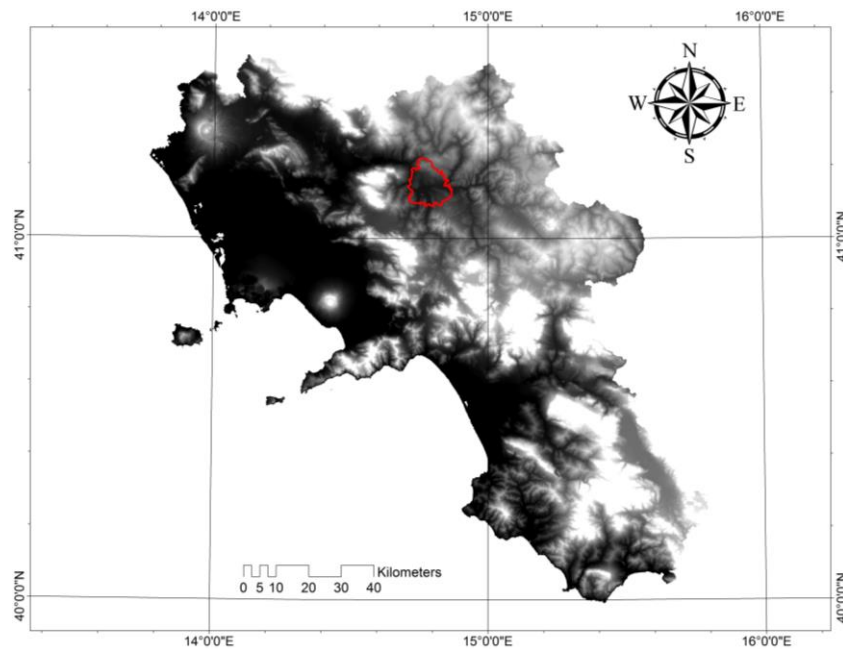


Fig. 2 Location of Benevento in Campania region

The approach is based on seven different combinations of GNSSs. These seven combinations are GPS; GLONASS; GALILEO; GPS + GLONASS; GPS + GALILEO; GLONASS + GALILEO; GPS + GLONASS + GALILEO. For the last combination, the presence of at least one satellite of each constellation is guaranteed, even if this may cause a higher GDOP value. For all seven solutions, each measurement session is performed with an interval of 15 minutes, for a total of 96 daily samples. The Ephemeris file is updated to May 18, 2016 whereas the simulations period is 1 day (May 19, 2016).

Different values are introduced for cut-off angle; they are set to 0° , 10° , 15° , 20° , 25° , 30° , in order to consider interference problems caused by buildings and trees and multipath errors. According to Mosavi [33], GDOP quality can be classified in: Good if $1 \leq \text{GDOP} < 3$; Fair if $3 \leq \text{GDOP} < 5$; Bad if $\text{GDOP} \geq 5$. In Tables 2-7, GDOP quality is reported for each constellation and cut-off angle, considering the percent distribution of the 96 samples in reference to the previous three classes (Good, Fair, Bad) plus a null class (when GDOP is unavailable because of insufficient number of visible satellites).

Compared to the other GNSS constellations, GPS gives the best performance: until a cut-off elevation angle of 15 degrees, its positioning service is trustworthy. For cut-off angle equal or greater than 20 degrees, the positioning accuracy classified as

good dramatically drops down from 48 percent (at 20°) to 0 percent (at 30°). In the GNSS context, the GLONASS constellation represents the second better choice: it gains a sufficient positioning precision. The Russian GNSS has good performance until a cut-off angle of 10°: in fact, after this value, its performances start to deteriorate much faster than GPS.

The comparison between GPS and GLONASS highlights how the difference in accuracy tends to increase reaching for the class good the maximum value (40%) at degree 15: this percentage value is quite rapidly reduced incrementing the cut-off angle. The GALILEO constellation is an unreliable solution because it presents total satellites unavailability for cut-off angles equal or greater than 15°. Only for 0°, the European GNSS reaches for the class good of position accuracy the 33 percent. For what concerns the GNSS combinations with the presence of cut-off angles, the GPS+GALILEO best quality percentage varies between 0% and 97%, while the GLONASS+GALILEO values are about 0%, 70%. The results show that GPS+GLONASS solution presents improved results over than GPS+GALILEO and GLONASS+GALILEO solutions: these enhancements are especially noticeable for the cut-off angle values equal or greater than 15 degrees.

Table 2 GDOP quality for a cut-off angle of 0°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	100	0	0	0
GLONASS	99	1	0	0
GALILEO	33	32	20	15
GPS+GLONASS	100	0	0	0
GPS+GALILEO				
GLONASS+GALILEO				
GPS+GLONASS+GALILEO				

Table 3 GDOP quality for a cut-off angle of 10°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	93	7	0	0
GLONASS	65	33	1	1
GALILEO	7	22	32	39
GPS+GLONASS	99	1	0	0
GPS+GALILEO	97	3	0	0
GLONASS+GALILEO	70	29	1	0
GPS+GLONASS+GALILEO	100	0	0	0

Table 4 GDOP quality for a cut-off angle of 15°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	82	17	1	0
GLONASS	42	47	9	2
GALILEO	0	13	31	56
GPS+GLONASS	96	4	0	0
GPS+GALILEO	83	16	1	0
GLONASS+GALILEO	44	51	5	0
GPS+GLONASS+GALILEO	96	4	0	0

Table 5 GDOP quality for a cut-off angle of 20°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	48	44	8	0
GLONASS	23	31	41	5
GALILEO	0	4	22	74
GPS+GLONASS	69	31	0	0
GPS+GALILEO	48	45	7	0
GLONASS+GALILEO	16	50	34	0
GPS+GLONASS+GALILEO	64	35	1	0

Table 6 GDOP quality for a cut-off angle of 25°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	11	61	28	0
GLONASS	9	21	61	9
GALILEO	0	2	13	85
GPS+GLONASS	26	63	11	0
GPS+GALILEO	12	63	25	0
GLONASS+GALILEO	4	36	60	0
GPS+GLONASS+GALILEO	19	71	10	0

Table 7 GDOP quality for a cut-off angle of 30°

	GDOP Quality (%)			
	Good	Fair	Bad	Null
GPS	0	37	59	4
GLONASS	0	8	76	16
GALILEO	0	0	11	89
GPS+GLONASS	3	57	40	0
GPS+GALILEO	0	43	56	1
GLONASS+GALILEO	0	11	88	1
GPS+GLONASS+GALILEO	1	63	36	0

Considering a cut-off angle of 30° (Fig. 3), the single constellations never achieve the maximum quality value. The best class value gained by GPS and GLONASS is fair that is reached respectively for 37% and 8 % of the daily observations. Galileo has the worst performance in absolute: for the 89% of the measurements the number of the available satellites is insufficient and for the remaining 11 % the GDOP class is bad. For a better positioning performance, it is necessary combining at least two GNSS constellations. In these scenarios, an integrated GPS+GLONASS solution permits to achieve the higher percentage (3%) of the best GDOP quality. GPS+GALILEO combination is relatively stable whereas GLONASS+GALILEO solution hardly deteriorates the positioning accuracy. For this case, Figs. 4-6 report satellites visibility charts for the single GNSS.

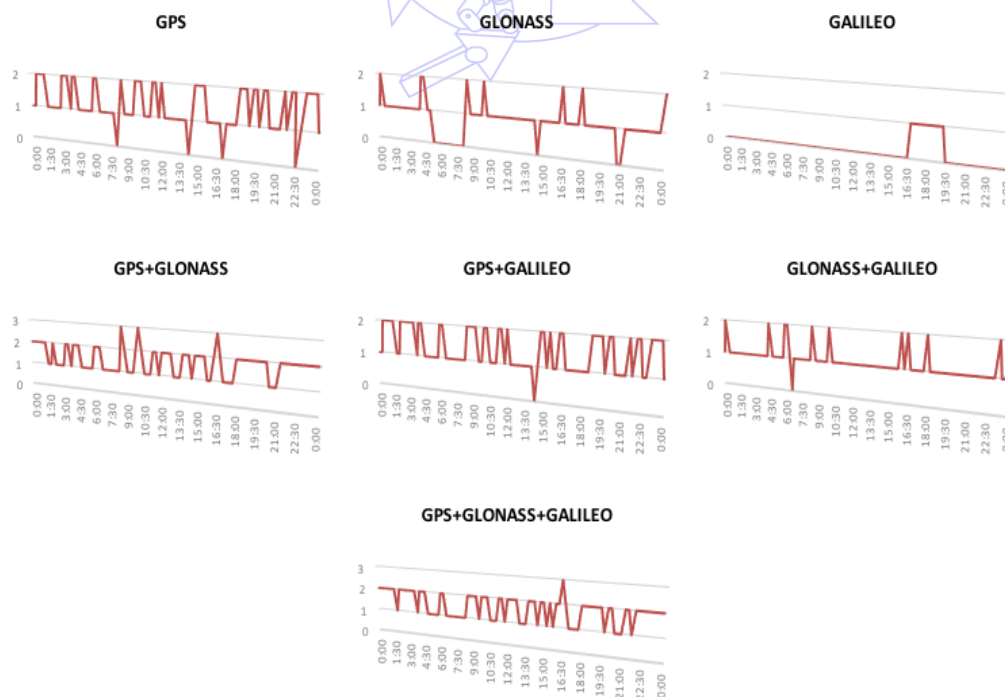


Fig. 3 GDOP quality distribution for a cut-off angle of 30°

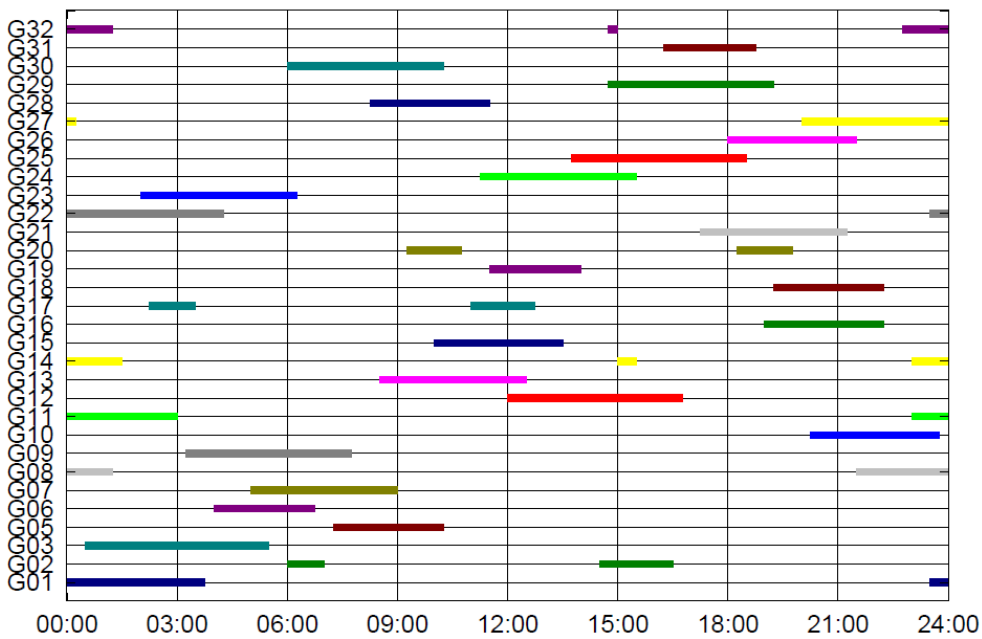


Fig. 4 GPS satellites visibility predictions for a cut-off angle of 30°

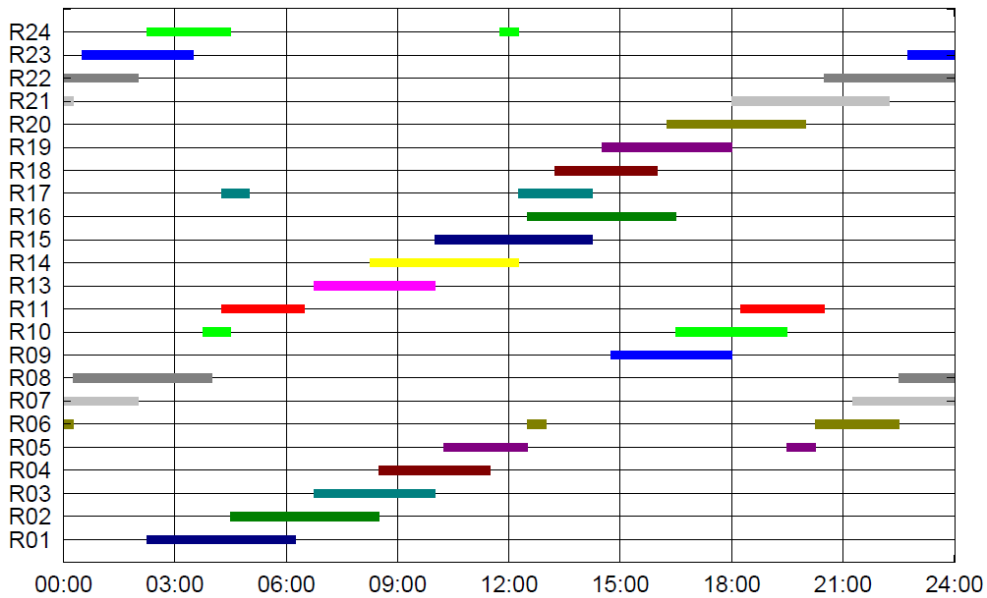


Fig. 5 GLONASS satellites visibility predictions for a cut-off angle of 30°

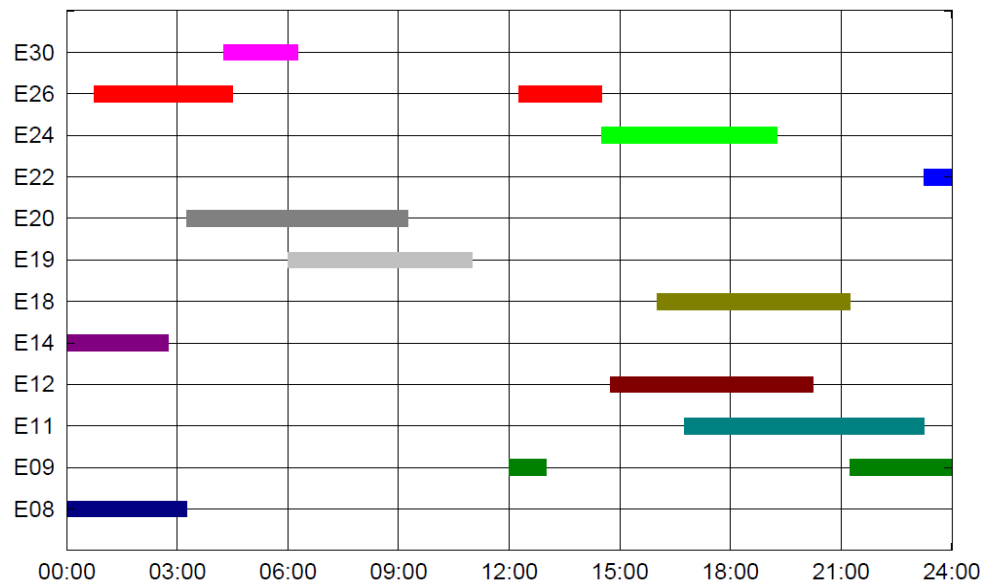


Fig. 6 GALILEO satellites visibility predictions for a cut-off angle of 30°

4. Conclusion

With the new and emerging constellations, satellite navigation is quickly changing: more satellites in view transmit navigation data at more frequencies. Currently, four GNSS constellations are full or partial operative: GPS, BeiDou Navigation Satellite System, GLONASS and GALILEO. Today professional receivers capable of receiving signals from these four constellations are available.

To establish a basis of evaluation and comparison, in this paper, several GNSS integration solutions are considered. The highest improvement due to the joint utilization of various GNSS regards the urban applications. Their main benefits consist of a better accuracy, reliability and availability, particularly evident on places with reduced sky visibility.

The multi constellations enhancements are noticeable in a wide range of applications for positioning, navigation and timing services compared to single constellation solutions. They are due to the increased redundancy provided by the GNSS combination.

For the multi GNSS constellations, GDOP is a significant criterion for satellite selection and evaluation of positioning accuracy. Thus, GDOP values are analyzed in this paper to confirm the increment of the position accuracy due to the integration of GNSS constellations.

Looking at the results, the improvements derived from these combinations are more evident using several cut-off angles, ranging from 0° to 30° . Multi GNSS solutions bring strong enhancements over the use of constellations alone: particularly, GPS+GLONASS supplies higher performances compared to the other solutions based on two constellations combination. It happens because of the incomplete character of GALILEO constellation: due to the insufficient number of satellites in view, it is not able to perform an autonomous positioning and its contribution and its presence instead of GPS or GLONASS in the multi-GNSS solutions produces a decrease in positioning accuracy. In addition, the presence of GALILEO in combination with GPS and GLONASS reduces in any case the best positioning performance if compared with the corresponding GPS+GLONASS; in fact the presence of at least one GALILEO satellite may lead to an impoverishment of geometry.

Furthermore, BeiDou Navigation Satellite System could be incorporated in this kind of study. Future research prospects are oriented to extend this comparison including BeiDou Navigation Satellite System in multi-GNSS combinations.

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