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# Intelligent Correction and Monitoring of Ship Propulsion Rotary Device Vibration

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## Abstract

Field inspection is a traditional way to detect the problem of shaft imbalance or abnormal vibration in a ship propulsion system; however, the ship cannot execute any tasks or activities during calibration. This study develops a human-machine monitoring interface (HMMI) to estimate vibration abnormalities and implement an intelligent active balance correction to the propulsion system online. In this study, Arduino IDE, InduSoft, and LabVIEW are used to create a function monitored by HMMI. By comparing the abnormal vibration amplification of the moment of inertia, HMMI calculates the correct mass to reduce the vibration. The experimental results show that, after HMMI carries out continuous active balance correction online, the correction rate achieves 105.37%. This indicates that HMMI can calculate the amount of imbalance and phase angles and drive a counterweight to the correct balance position while the device is still operating.

**Keywords:** Arduino IDE, InduSoft, HMMI, LabVIEW, active balance

## 1. Introduction

Timely detection and correction can prevent the worsening of ship propulsion systems, and further damage to parts of the propulsion rotary devices can be avoided. Thus, balancing motors is a crucial process for ship propulsion systems. This balancing work has been mostly done by field experts using offline methods. However, the offline methods are time-consuming and inefficient that cannot address the issue of changing imbalance distribution during system operation.

The published literature on automatic balance correction in the past has primarily used the counterweight method for balance compensation. However, this method requires a constant start/stop testing and balance weight adjustment on the rotating device, which wastes significant time and manpower.

In this study, a human-machine monitoring interface (HMMI) is developed to monitor abnormal vibration and balance correction of a ship propulsion rotary device online (as shown in Fig. 1). Through the comparison method of amplitude and phase angles, the balance weight adjustment can be carried out without stopping the device, achieving the objective of online balance correction. To verify the accuracy of HMMI, a commercial advanced vibration analyzer of VMI-VIBER X5 PRO (ISO 10816) is used. The root mean square (RMS), phases, and cycles per minute (CPM) of rotors are approximately 0.06~0.16%, 0.12~0.26%, and 0.08~0.12%, respectively. A self-balancing physical platform is built to verify the HMMI application to intelligent and active balance correction online.

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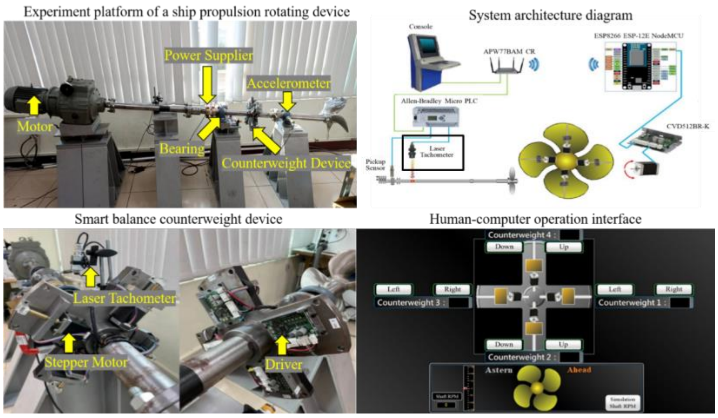


Fig. 1 Vibration monitoring and balance correction for ship propulsion rotating devices

The novel aspect of the study lies in the fact that it enables rapid and automatic correction of vibration while the equipment is in operation, thus stopping the impact of imbalance from spreading further. It thereby ensures the stability and safety of the rotary device and prevents abrupt malfunctions. In addition, regular crew members can easily understand the current status of the ship's propulsion system through HMMI.

## 2. Literature Review

A propulsion system is a critical rotary component of a ship that provides thrust to move it through the water. Just like any other rotary system, abnormal vibration is fairly common in ships' propulsion systems that often result due to dynamic imbalance [1-2]. In the mid-19th century, more flexible rotors were introduced for different types of machinery, which further increased the demand for improved dynamic imbalance methods. In 1964, the least square method was introduced, which was essentially based on the influence coefficient method (ICM) and was aimed at solving the contradiction between measure points and balance planes [3]. The point speed ICM was introduced in 1972 by Tessarzik, which aimed at balancing rotating machinery [4]. Three different conditions of initial rotor unbalance were used to evaluate the method, and rotors were successfully balanced. In 1975, Tessarzik further extended his rotor balancing work and conducted experiments to demonstrate that the ICM can achieve precise balance for any system, running at any speed [5].

In 1976, a linear programming approach was introduced for balancing flexible motors in which discrete components of the unbalanced rotor were identified and removed using constraints to keep the weights within the safe limits [6]. In 1978, an online automatic balancing method for rigid rotors was proposed, in which sets of counterweights are placed on each side of the rotating machinery, and the counterweight is moved by electromagnetic control to suppress vibration [7]. Further work by Van de Vegte et al. [8] provided a theoretical framework for the potential application of automatic balancing for flexible motors. In 1987, Lee et al. [9-10] extended this concept to flexible rotors. They controlled a counterweight on the axial to reduce the abnormal vibration of the rotors and designed a system that automatically balanced the device by controlling electromagnetic pulses to drive the counterweight to a point of balance. Further insight into different balancing procedures for rigid and flexible shafts can be gained from a 1991 review article by Parkinson [11]. A modified version of the ICM was introduced in 1996, based on the complex co-ordinate representation and finite element model, for balancing unsymmetrical rotor-bearing systems [12].

The automatic ICM was successful in controlling imbalance; however, if initial influences are inaccurately estimated, the vibration may worsen even further before converging to the point of balance. To counter this problem, an auto-tuning method was introduced in 2000 that controlled the adaptive control parameters of convergence while a supervisory control system selected vibration limits to control any unexpected vibration [13]. A 2001 review article by Zhou et al. [14] discussed dynamic modeling and analysis techniques of rotor systems to achieve active balancing. They noted that active balancing could reduce the dynamic imbalance and thus improve the machine life, but further work needs to be done from a cost-effective point of view to reduce the system cost [14].

In 2003, Wang et al. [15] presented an electromagnetic online automatic balancing system and discussed its principle, structure, and balancing method. In the same year, Tonnesen [16] used the ICM to create an experimental device with unbalanced counterweights to simulate abnormal rotor vibration. The device was capable of observing and comparing unbalanced errors and normal positions. In 2005, Lee et al. [17] proposed an electromagnetic active balancing device and control programs. The methods were tested on the real spindle system for machine tools in practice. The influence coefficient of a reference model was used as the gain. This method adds known counterweights to both sides of a rotor and sets up electromagnetic balancing wheels. The balancing position is estimated using the ICM, and balancing wheels achieve a balanced reduction in vibration. They also successfully carried out balancing experiments with varying operating speeds with a continuous spindle rotation.

In 2006, Hredzak and Guo [18] inscribed multiple radial grooves around the axis of rotation on the surface of a disc-like device, and then placed freely-moving steel balls into each groove. A sensor was used to locate the equilibrium position. Orientation and balance calculations determined the balance position of the balls, and electromagnetic pulses drove them to the correct balance position. Experimental results indicated that the proposed device can reduce the rotational imbalance even when the value and position of imbalance are changing over time. The introduction of new technologies in rotor systems continued, and in 2008, Untaroiu et al. [19] introduced a convex optimization method along with ICM to flexible motors. This method utilizes the inertial force generated by the calibration weight to counteract the vibration caused by the unbalanced rotor. By measuring the initial vibration state and weighted vibration state of different balance surfaces, the influence coefficient matrix and correction weight can be determined, and the balance correction can be performed accordingly.

In 2015, an imperialist competitive algorithm in conjunction with ICM was introduced by Mohammadi et al. [20] to balance flexible motors. A 2016 review article discussed advances in propulsion technology and proposed the use of metal rubber technology in thrust bearing [21]. In 2017, Fu et al. [22] and Huang et al. [23] used the transient balancing method to balance the rotor system. In 2021, Chai and Wu [24] used a novel hybrid method to analyze the longitudinal excitation of the propeller and judge whether the amplitude is within the allowable range. Deng et al. [25] used the transient dynamic balancing method without trial weights, and the ICM was used to balance a power turbine rotor by numerical simulation. The transient dynamic balancing method without trial weights was found to have a better balancing effect than other balancing methods [25]. Li et al. [26] discussed traditional balancing methods such as ICM and modal balancing method (MBM), and the classification of new balancing methods was reviewed.

## 3. Methods

### *3.1. Concept design and imbalance analysis*

The preliminary phase of this study was completed on a simple experimental platform (Fig. 2) with the proposed HMMI. The LabView-designed HMMI was used in the simulations to retrieve vibration signals pertaining to the rotary speed, amplitude, and phase angle (Table 1) of a direct current fan operating.

The accuracy of the signals was validated through an advanced vibration analyzer of VMI-VIBER X5 PRO (Fig. 3) in accordance with ISO10816 standards, as well as the characteristics of the vibration generated by counterweights on the rotary device. The experimental results suggested that the error between the signals retrieved by the proposed HMMI and the advanced vibration analyzer of VMI-VIBER X5 PRO was less than 1.5%, in accordance with ISO10816 standards.

According to the data of the vibration generated by counterweights ranging from 6 g to 35 g on the rotary device, the amplitudes along the x-axis (radially left and right) and the y-axis (radially front and back) had greater variations than those along the z-axis (axial direction). In terms of different rotary speeds, the amplitude variations along the y-axis (radially left and right) were more prominent (Figs. 4-6).

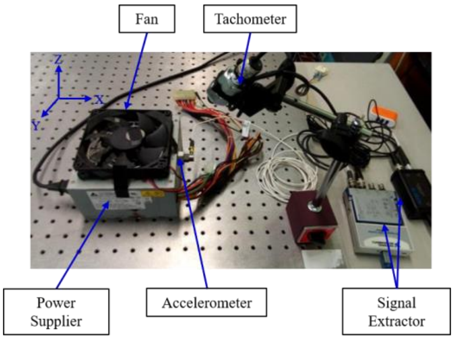


Fig. 2 The simple experimental platform designed in the preliminary phase

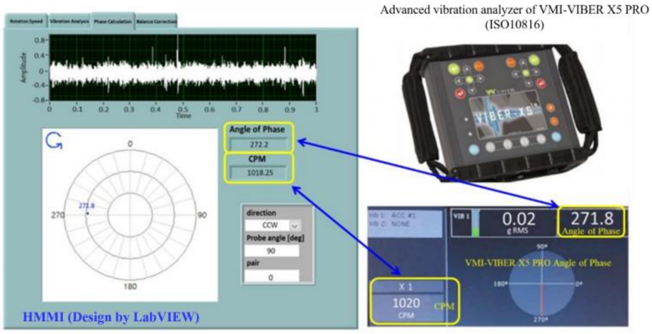


Fig. 3 Schematic of signal accuracy validation

Table 1 Validation data of signal accuracy

| Data type | VMI-VIBER X5 PRO | HMMI | Error |
|---|---|---|---|
| Amplitude (mm/s RMS) | 0.54 | 0.533 | 1.3% |
| Phase angle (°) | 271.8 | 272.2 | 0.1% |
| Rotary speed (CPM) | 1020 | 1018.25 | 0.2% |

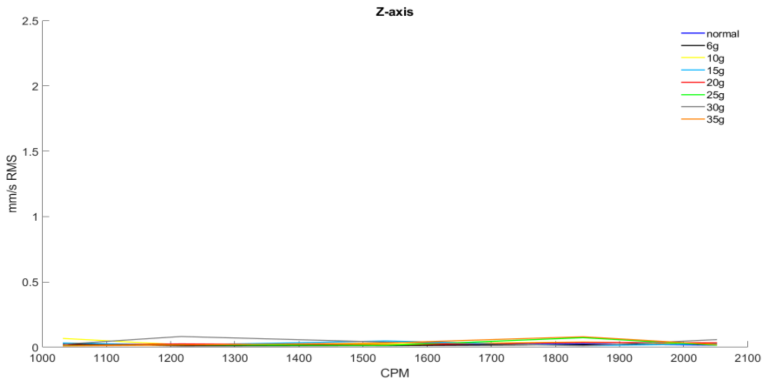


Fig. 4 Relationship between the amplitude and rotary speed along the z-axis (axial direction) (mm/s RMS－CPM)

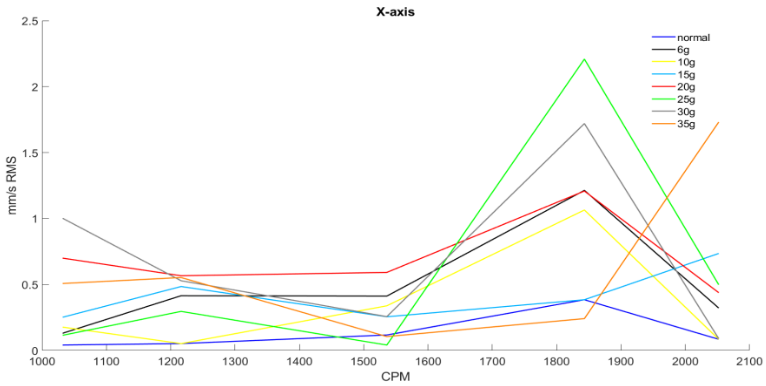


Fig. 5 Relationship between the amplitude and rotary speed along the x-axis (radially left and right) (mm/s RMS－CPM)

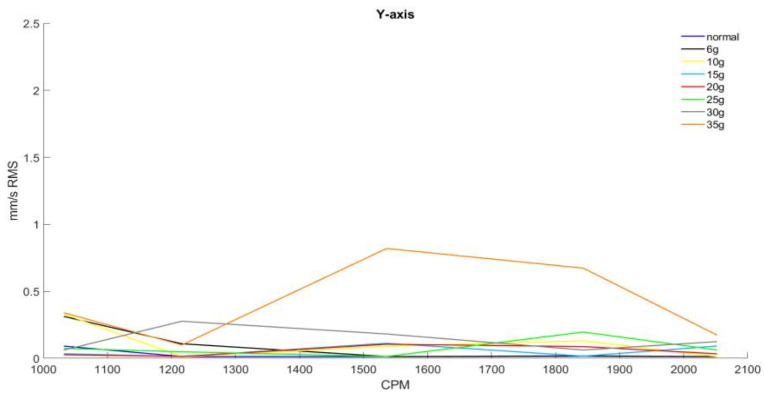


Fig. 6 Relationship between the amplitude and rotary speed along the y-axis (radially front and back) (mm/s RMS－CPM)

### 3.2. Initial design and active correction verification

The study developed an intelligent balancing system that consists of a signal sensor, a counterweight monitor, and an HMMI system for monitoring the abnormal vibration and recording the vibration spectra of a ship's propulsion device. The system automatically calculates the required counterweight and phase change. It drives a stepper motor to perform automatic balancing correction tasks online (Fig. 7), redress abnormal vibration, stabilize the operation of the propulsion system, and prevent further and progressive vibration-induced damage. The abnormal spectrum signals recorded by the system can serve as a reference for engineers to swiftly identify and replace malfunctioned components and reduce maintenance times when a ship is docked. To validate the feasibility of the automatic balancing system, the study also included tapped holes along the outer rim of the circular load such that different counterweights can be added to examine whether the automatic balancing system and the HMMI can allocate counterweights and achieve automatic balancing and correction.

The mathematical model is based on the moment of inertia $I = \int \rho r^2 \, dV$, where $\rho$ is the density and $dV$ is the infinitesimal volume change causing the ship propulsion rotary imbalance. The torque $T = I\alpha$ occurs during the operation of the ship propulsion rotary device, where $\alpha$ is the angular acceleration. Therefore, if the ship propulsion system's rotating device sustains damage to the shafting, propeller, reduction gear, or bearing, the resulting dynamic imbalance will cause a change in the moment of inertia $I$ and produce abnormal vibration. If a restrained balance weight can be generated at the phase angle at the other end, the expansion damage caused by the abnormal vibration can be limited, and the reliability of the ship system can be improved. The general principle function is:

$$\frac{I_{system}}{dt} = \frac{I_{abnormal}}{dt} - \frac{I_{initial}}{dt} \tag{1}$$

$$\frac{I_{correction}}{dt} = \frac{I_{counterweight}}{dt} - \frac{I_{initial}}{dt} \tag{2}$$

The ship's rotary propulsion system is balanced when the value of counterweight equals the amount of the system's imbalance.

$$\frac{I_{counterweight}}{dt} = \frac{I_{system}}{dt} \tag{3}$$

From Eqs. (4)-(6), Eq. (7) can be obtained.

$$T = I\alpha \tag{4}$$

$$\alpha = \frac{d\omega}{dt} \tag{5}$$

$$T = mgr \tag{6}$$

$$mgr = I\frac{d}{dt}\omega = I\frac{d}{dt}\left(\frac{d\theta}{dt}\right) \tag{7}$$

After securing a 6 g counterweight onto the outer rim of the circular load at an angle of 150°, the HMMI indicated that the system's vibration increased from 0.455 RMS to 0.622 RMS. Following the first automatic balancing correction, the counterweights were moved to reduce the stabilized system's vibration from 0.622 RMS to 0.465 RMS. Afterward, the second automatic balancing correction was performed to reduce the system's vibration to 0.440 RMS. The experimental data is shown in Table 2. The first and second automatic balancing corrections had an accuracy of 98% and 103%, respectively.

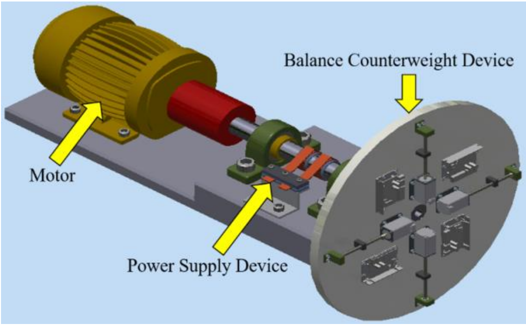


Fig. 7 Prototype of the automatic balance correction system

Table 2 Active correction data of the proposed system

| Data type | Initial state | Locked at 150°, 6 g weight imbalance | First correction | Second correction | Correction rate |
|---|---|---|---|---|---|
| Vibration (mm/s RMS) | 0.455 | 0.622 | 0.465 | 0.440 | 103% |
| Phase angle (°) | 41.25 | 140 | 38.2 | 40.3 | 97% |

### 3.3. System prototype development and verification

The study used InduSoft, Arduino IDE, and LabVIEW to create a function monitored by HMMI. A weight was attached to one side of a propeller to simulate unbalanced abnormal vibration (Fig. 8). Once the propeller was running, the advanced vibration analyzer of VMI-VIBER X5 PRO carried out synchronous measurements, while the HMMI displayed rotor CPM, RMS, and phase (Figs. 9-10). Table 3 shows the error values for a 220 g weight. The experimental platform consisted of a Wi-Fi device, Micro PLC, pickup sensor, NodeMCU, CVD512BR-K driver, and stepper motor (Fig. 11), and the hardware specifications are shown in Table 4.

Fig. 8 Weights attached to a propeller to simulate imbalance

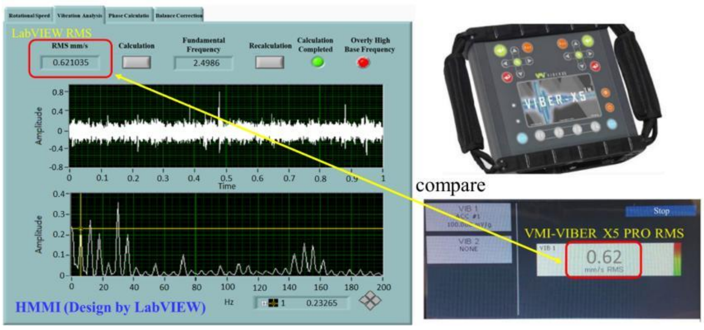


Fig. 9 Comparison of the RMS value obtained from HMMI and VMI-VIBER X5 PRO

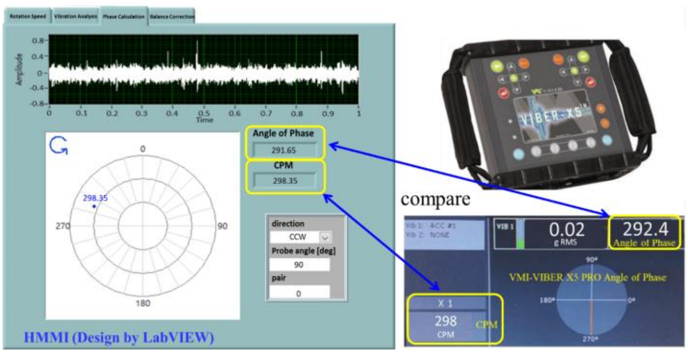


Fig. 10 Comparison of the CPM and phase angle values obtained from HMMI and VMI-VIBER X5 PRO

Table 3 HMMI and VMI-VIBER X5 PRO data comparison and analysis

| HMMI and VMI-VIBER X5 PRO data comparison and analysis | | | |
|---|---|---|---|
| Data type | LabVIEW | VMI-VIBER X5 PRO | Error ratio (%) |
| Vibration (mm/s RMS) | 0.621 | 0.62 | 0.16 |
| Phase angle (°) | 291.65 | 292.4 | 0.26 |
| Rotary speed (CPM) | 298.35 | 298 | 0.12 |

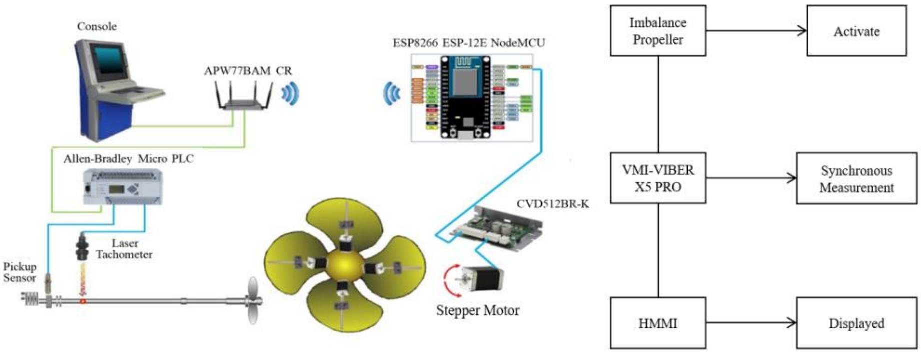


Fig. 11 System structure of the experimental simulation platform

Table 4 Hardware specification

| No. | Name | Specification |
|---|---|---|
| 1 | Motor | Model no.: EBFC-D (3 HP)<br>Rotary speed: 1-1400 RPM |
| 2 | Bearing | Model no.: UC210-32<br>d: 50.8 mm<br>L: 206 mm<br>H2: 57.2 mm |
| 3 | Stepper motor | Model no.: PKP525N12A<br>Rated power: 1.2 A<br>Basic step angle: 0.72° |
| 4 | Driver | Model no.: CVD512BR-K 5-phase stepper motor driver<br>Power: DC 24 V ±10% 1.7 A |
| 5 | Development board | Model no.: NodeMCU (802.11 b/g/n standard)<br>Input power: 4.5-9 V (10 VMAX)<br>Transmission rate: 110-460800 bps |
| 6 | Counterweight device | Lead screw: 1.2 mm<br>Slider weight: 9 g<br>Smallest distance of movement: 1.2 / (360 / 0.72) = 0.0024 mm |
| 7 | Power supplier | Model no.: LRS-75-24<br>Rated power: 76.8 W<br>Input power: 1.4 A / 115 VAC, 0.85 A / 230 VAC<br>Output voltage: 21.6-28.8 V<br>Output current: 3.2 A |
| 8 | Homemade voltage regulator | Uses 7805A regulator IC<br>Input power: 24 VDC<br>Output power: 5 VDC |
| 9 | Power supply device | Adopts conductor lapped power supply; requires supply to the stepper motor, driver, and development board |
| 10 | Accelerometer | Model no.: IEPE Accelerometer<br>Sensitivity: 100 mV/g<br>Integrated circuit piezoelectric |
| 11 | Laser tachometer | Model no.: ROLS-W<br>Scope of measurement: 1-250000 RPM<br>Output signal: TTL signal |
| 12 | Signal capture card | Model no.: NI-9234<br>Highest sampling rate: 51.2 ks/s<br>Analog input voltage range: -5 V to 5 V |
| 13 | Advanced vibration analyzer (ISO 10816) | Model no.: VMI-VIBER X5 PRO<br>Frequency range: 0.5-25600 Hz<br>Vibration range: 0-80 g |

When the system is activated, the tachometer and the pickup sensor will measure the current operational state of the axis of the propulsion system, while transmitting the rotary speed, vibrational frequency, amplitude, and phase angle back into the HMMI at the control panel. The HMMI then calibrates and monitors the system and calculates the counterweights required to balance out the imbalanced and abnormal vibration. The signals are transmitted over Wi-Fi to the NodeMCU, which drives the stepper motor through the CVD512BR-K driver to balance out the weights and stabilize the propulsion system. The HMMI displayed and recorded signals for the rotary speed, vibration, and phase angle and compared these with the initial amplitude of the system. By using this feedback and calculation, the abnormal vibration warning module displayed warnings of ongoing vibration abnormality, drove a stepping motor to move counterweights to a balanced position, made intelligent balance corrections, and recorded abnormal spectrum data for component failure analysis.

## 4. Results and Discussion

### *4.1. Initial state*

The advanced vibration analyzer of VMI-VIBER X5 PRO (ISO 10816) was used to verify the accuracy of the experimental platform. After the comparison with the initial amplitude at a speed of 299.55 CPM, the RMS amplitude was 0.159 mm/s, the base frequency was 5.00608 Hz, and the phase angle was 36.45° (Fig. 12).

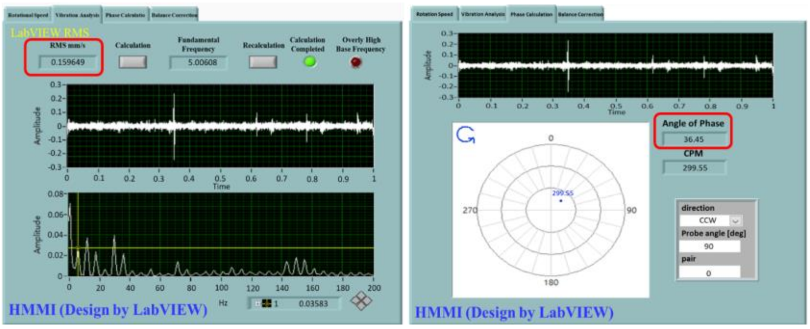

Fig. 12 Initial state of the experimental platform

### *4.2. Simulated imbalance caused by 66 g oysters attached to the propeller*

With simulated 66 g weight imbalance, the HMMI showed that RMS amplitude increased to 0.236 mm/s (Fig. 13), and the phase angle increased to 233.55° (Fig. 14). The counterweight 1 moved 30 steps, and the RMS and phase changed to 0.208 mm/s and 128.12°, respectively (Fig. 15). The counterweight 4 moved 35 steps, and the RMS and phase changed to 0.163 mm/s and 37.14°, respectively (Fig. 16). Data (Table 5) shows a balance correction rate of 96.32% for RMS and 98.14% for the phase angle.

**Apply 66 g weight imbalance on the propeller**

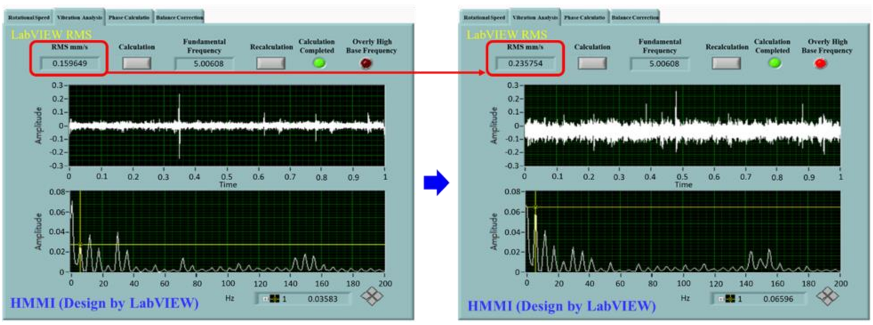

Fig. 13 RMS change with 66 g weight imbalance

**Apply 66 g weight imbalance on the propeller**

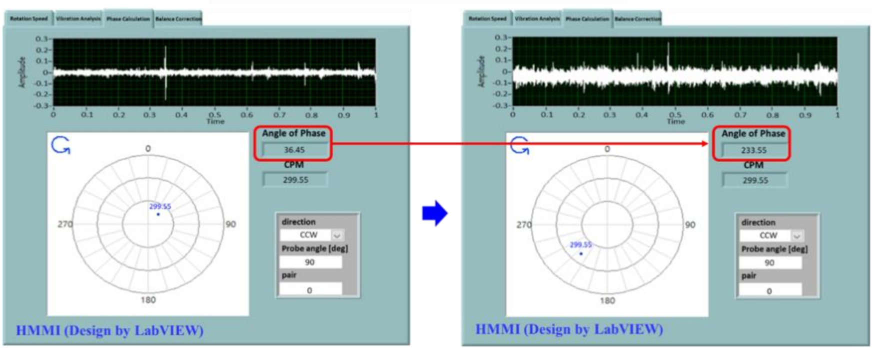

Fig. 14 Phase angle change with 66 g weight imbalance

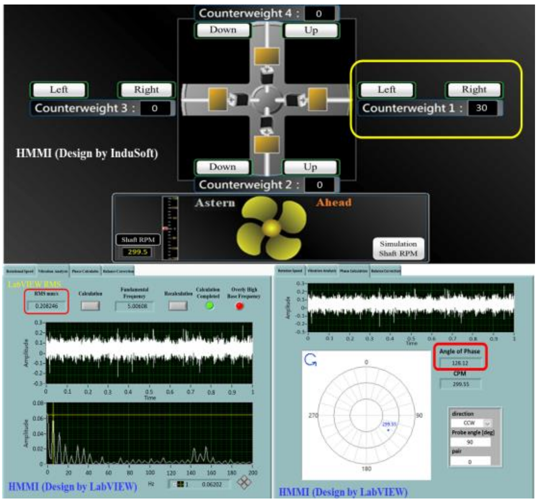


Fig. 15 Counterweight 1 moves 30 steps for system imbalance correction

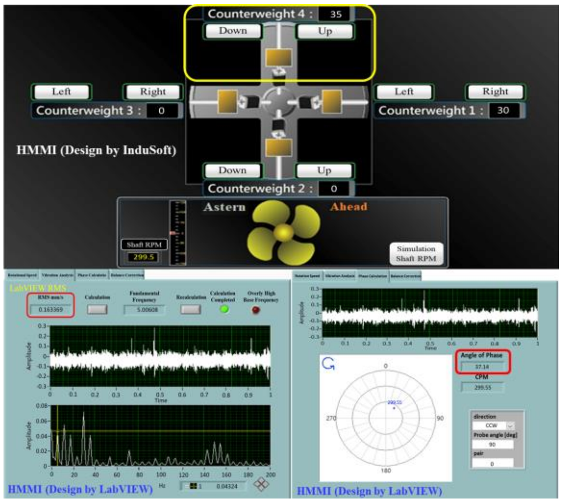


Fig. 16 Counterweight 4 moves 35 steps for system imbalance correction

Table 5 Signal data of vibration and phase angle

| Data type | Initial state | Locked at 225° 66 g weight imbalance | Counterweight 1 (30 steps) | Counterweight 4 (35 steps) | Correction rate |
|---|---|---|---|---|---|
| Vibration (mm/s RMS) | 0.159 | 0.236 | 0.208 | 0.163 | 96.32% |
| Phase angle (°) | 36.45 | 233.55 | 128.12 | 37.14 | 98.14% |

### *4.3. Simulated imbalance caused by 220 g oysters attached to the propeller*

At a speed of 299.55 CPM, when the system simulated 220 g weight imbalance, the HMMI showed that RMS increased from 0.159 mm/s to 0.541 mm/s (Fig. 17), and the phase increased from 36.45° to 291.65° (Fig. 18). Following the first active correction, the RMS changed to 0.168 mm/s (Fig. 19). To further reduce the imbalance vibration, the system automatically carried out a second correction, and RMS changed to 0.149 mm/s (Fig. 20). Data (Table 6) shows a balance correction rate of 105.37% for RMS and 100.61% for the angle of phase.

**Apply 220 g weight imbalance on the propeller**

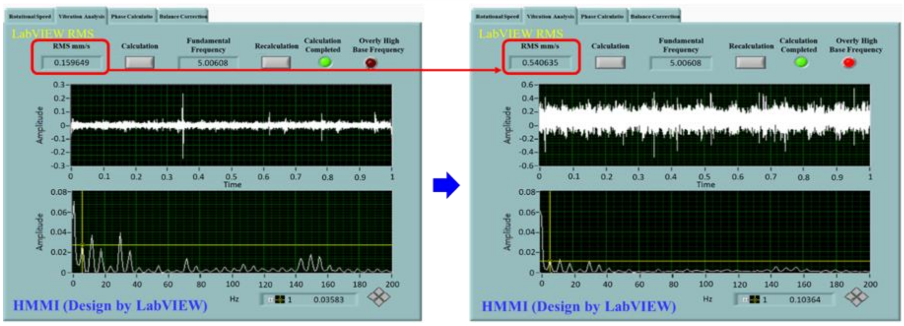


Fig. 17 RMS change with 220 g weight imbalance

**Apply 220 g weight imbalance on the propeller**

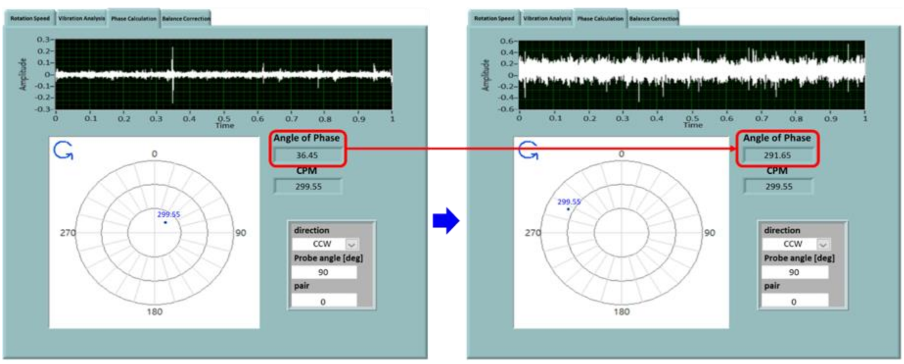


Fig. 18 Phase angle change with 220 g weight imbalance

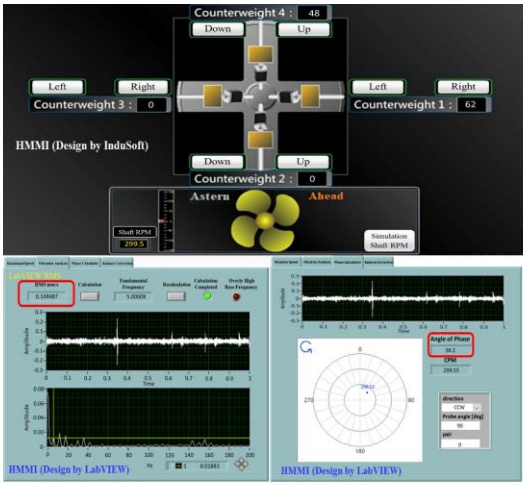

Fig. 19 First active correction online

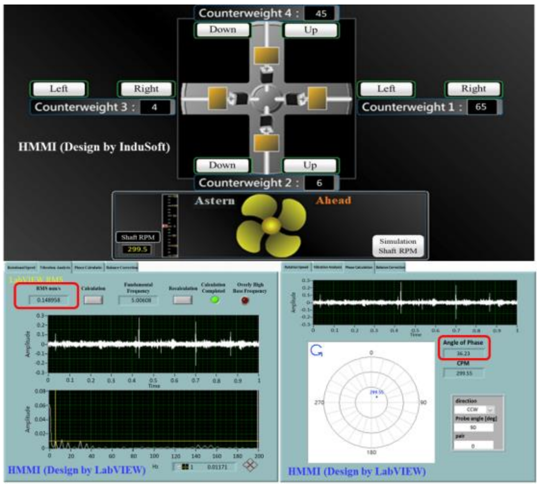

Fig. 20 Second active correction online

Table 6 Active correction data of the proposed system

| Data type | Initial state | Locked at 225° 220 g weight imbalance | First correction | Second correction | Correction rate |
|---|---|---|---|---|---|
| Vibration (mm/s RMS) | 0.159 | 0.541 | 0.168 | 0.149 | 105.37% |
| Phase angle (°) | 36.45 | 291.65 | 38.2 | 36.23 | 100.61% |

## 5. Conclusions

A ship propulsion system, like other rotating machinery, suffers from rotor imbalance. In the past, field experts manually monitored and corrected such imbalance, and offline methods were often used to control the imbalance; however, these methods suffered from methodological limitations and were not suitable for a dynamic system like a propulsion system. The limitations lead to an increase in studies in finding more efficient balancing methods, particularly for heavy-load and high-speed rotating machinery. This study developed an HMMI system to carry out balance correction online and conducted experiments and analyses to verify the accuracy of the proposed system. From the study, the following conclusions were made:

(1) The RMS, phase, and CPM error values were about 0.16%, 0.26%, and 0.12%, according to the comparison between the experimental results obtained from the proposed system and the advanced vibration analyzer (VMI-VIBER X5 PRO).

(2) The experiments confirmed that the proposed system can continuously carry out active balance correction online. After the first active correction online, the amplitude correction rate was 93.45%, and the angle of phase was 95.41%. Following the second correction, these values changed to 105.37% and 99.39%, respectively. Although there remained some residual vibration after the first correction, not only was this completely corrected by the second correction but the original imbalance was also corrected.

(3) The proposed system can prevent the impact of imbalance from causing serious failure in a ship's propulsion rotary device, and thus improve the reliability of the rotary device.

(4) Through the use of the proposed system, regular crew members can easily understand the current status of the ship's propulsion rotary device by observing the vibration signals and the four counterweight positions.

(5) The HMMI system proposed in this study shows better performance compared to those in most previous studies using constant start/stop testing and balance weight adjustment method.

## 6. Featured Application

The HMMI system can enhance the reliability of a rotary device. The calculations were carried out by HMMI to drive a counterweight to balance abnormal vibration and stop the impact of imbalance spread. If this system can be incorporated into the design of a ship's propulsion system console, regular crew members will easily understand the current status of the rotary device by observing the vibration signals and four counterweight positions. This will ensure the stable, safe, and effective operation of the device, and prevent serious failure due to abnormal vibration.

## 7. System Robustness and Future Work

If the locking mechanism (i.e., the rotary device) of a ship's propulsion system malfunctions, it can easily cause the system instability because the balancing counterweight can move back and forth continuously when adjusting the system imbalance. This situation will be prevented in the future by improving the locking mechanism and the filtering function of multi-frequency eigen-signal characteristics.

## Conflicts of Interest

The authors declare no conflicts of interest.

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