Proceedings of Engineering and Technology Innovation, vol. 28, 2024, pp. 67-80

Critical Assessment on the Stability and Convergence of the Conventional Gear Tooth Contact Analysis

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Received 27 July 2024; received in revised form 25 September 2024; accepted 27 September 2024

DOI: https://doi.org/10.46604/peti.2024.14062

Abstract

Mathematical modeling of gear engagement is crucial during design to ensure optimal performance in manufacturing. This study reproduces the conventional tooth contact analysis (TCA) model, highlighting convergence issues in parallel-axis gears and limitations in local synthesis methods. The research critically analyzes the TCA method, which solves five nonlinear equations to assess performance and accuracy. Simulations replicate the conditions of previous studies to ensure valid comparisons. Initial guess values are randomly generated within a specific range to guide the iterative process toward convergence, with this range progressively narrowed to improve computational efficiency and accuracy. Results indicate that the TCA approach is highly sensitive to initial guess values, particularly the starting angular position. Convergence issues arise from the complexity of nonlinear equations and multiple roots. This can lead to divergence or reverting to the initial guess when values deviate significantly from the true solution.

Keywords: gear TCA, convergences issue, misalignments, spur gear, helical gear

1. Introduction

Automotive gearing systems are fundamental to power transmission in vehicles, influencing overall performance and efficiency. Among the various methods for analyzing gear behavior, tooth contact analysis (TCA) is one of the most prominent techniques for studying the interactions between gear teeth. TCA helps engineers understand contact patterns, transmission errors, and stresses during meshing.

The development of accurate TCA models has become increasingly important due to the demand for more reliable and efficient automotive systems. The approach developed by Litvin and Fuentes [1] is the most established and widely adopted methodology for gear tooth meshing and surface contact. This approach addresses contact issues using the established surface tangency condition, leading to a set of nonlinear equations that typically require numerical solution methods. These equations form a system of five generally nonlinear equations with one independent and five unknown parameters. By utilizing this method, it is possible to estimate transmission errors and contact points throughout complete tooth mesh cycles. The relative curvatures of the contacting surfaces can be used with contact mechanics to calculate the instantaneous deformed geometry of the contact interface and its area [2-5].

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English language proofreader: Chih-Wen Teng

For several decades, this method has been widely used in gear contact analysis across various applications. For example, it has been employed to analyze low-noise, adjusted bearing contact spiral bevel gears [6] and to simulate meshing and stress analysis in helical gear drives [7-10]. Zschippang et al. [11] utilized this TCA model to efficiently generate the geometry of face gears, considering the finite length of the shaper. They also explored methods for crowning on face-gear flanks and calculating its effects. Jones et al. [12] used it to analyze the impact of misalignment on the contact pattern, load distribution, and tooth stresses in gear pairs. Liu et al. [13] applied Litvin's model to study tooth contact patterns and contact stresses in face-milled spiral bevel gears. Tsuji et al. [14] used it for the analysis and manufacture of large-sized straight bevel gears with equi-depth teeth. Similarly, many other researchers have applied this TCA model over the decades [15-17].

Despite its widespread acceptance, this conventional method has some limitations. Numerical computations can encounter convergence problems if the initial values, or "guess values," are not carefully chosen. These challenges involve producing approximate solutions, depending on numerous unknown variables, which can significantly increase computational complexity and require substantial processing power to accurately determine the surface contact location. To address these issues, Litvin and colleagues proposed strategies such as "local synthesis" and heuristic sweeps of the parametric solution space to identify suitable "guess values" and achieve convergence [18-19]. However, these approaches can be impractical and computationally intensive. Various researchers [20-27] have noted this issue.

To address these challenges, Spitas and Spitas [21] introduced a novel two-dimensional system of equations for tooth surface contact in spur gears. However, their approach was unable to fully account for the complex spatial geometry involved in gear meshing, particularly in cases of surface modifications and system misalignments. Sheveleva et al. [28] later developed an algorithm based on a grid representation of tooth surfaces, using nodal points and distance calculations to determine contact patterns. Despite this progress, the algorithm struggles to accurately pinpoint contact positions and cannot be applied to face gear drives, which require high precision in contact point determination. Similarly, the kinematic geometry synthesis for pure-rolling contact spiral bevel gears, proposed by Zhang, et al. [29], is limited to pure-rolling gear drives and cannot be extended to face gear drives.

In recent years, researchers have explored new methods for analyzing gear tooth contact, offering more efficient alternatives to the conventional TCA model. For example, Wang et al. [30] proposed a digital TCA method that streamlines the process by utilizing an initial point algorithm, reducing the classic TCA model from five nonlinear equations with five unknowns to three scalar equations with three variables. However, this method still depends on a discretized approach, leading to approximate solutions. This creates challenges when dealing with complex tooth surfaces that feature extensive modifications and misalignments. Additionally, the use of a particle swarm optimization algorithm to identify the initial contact further increases the computational load.

To date, there have been no independent assessments or comprehensive reports on Litvin's basic model, despite its extensive use in various applications. This study aims to fill this gap a using the same approach and parameters employed by Litvin and colleagues to evaluate the model's numerical stability, accuracy, and sensitivity to initial conditions. The gear tooth surface contact equations proposed by Litvin will be referred to as the conventional TCA method. Rigorous analysis and simulations will be conducted to gain insights into the model's performance, limitations, and robustness. Additionally, the effects of varying initial conditions on the model's outcomes will be examined.

In this study, Section 1 provides an overview of the literature on TCA methods, discussing the background and identifying the research gap in conventional TCA methods. Section 2 reviews the mathematical framework of the conventional TCA model, emphasizing its core assumptions and computational methodologies. Section 3 presents the computational replication and evaluation of the model, along with a detailed discussion of the results for both aligned and misaligned gears. Section 4 concludes with insights from a comprehensive assessment of conventional TCA methods, evaluating their efficiency and accuracy.

2. Conventional TCA Model

In the scope of gear tooth analysis [1-8], the conventional TCA model consistently demonstrated a solution to the problem of the geometrical contact of two rotating surfaces, Fig. 1. Assuming the existence and continuity of surface gradients, the gear tooth contact is presented using a surface tangency condition. The tooth surfaces S_1 and S_2 are in point tangency, and vector equations in the coordinate system S_f represent the instantaneous tangency of surfaces.

$$\vec{r}_1^{\,f} - \vec{r}_2^{\,f} = 0 \tag{1}$$

$$\vec{n}_1^{\,f} - \vec{n}_2^{\,f} = 0 \tag{2}$$

where $\vec{n}_{1,2}^{\,f}$ represents the unit vector of the surface normal and $\vec{r}_{1,2}^{\,f}$ illustrates position vectors. Three independent scalar equations are produced by the vector equation Eq. (1), whereas only two are produced by Eq. (2).

$$\left|\vec{n}_1^{\,f}\right| = \left|\vec{n}_2^{\,f}\right| = 1 \tag{3}$$

If necessary, the collinearity of the surface normals can be found as follows:

$$N_1 = \lambda N_2,\ (\lambda \neq 0) \tag{4}$$

Considering convention gear 1 as a reference, Eqs. (1)-(2) give a set of six nonlinear equations, of which five are independent, with five unknowns and one applied φ_1 parameter.

$$f_i\left(u_1, v_1, \phi_1, u_2, v_2, \phi_2\right) = 0,\ f_i \in C^1,\ \ (i = 1, \ldots, 5) \tag{5}$$

The parameterization used is that u_1,v_1 and u_2,v_2 are any two parameters describing the corresponding surfaces in a local frame of reference bound to the respective gear, which rotates together with the gear by corresponding angles φ_1 and φ_2. Eq. (5) can be solved for a contact point following the Theorem of Implicit Function System Existence with the condition that the Jacobian is non-zero:

$$0 \neq \frac{D\left(f_1, f_2, f_3, f_4, f_5\right)}{D\left(u_1, v_1, v_2, \varphi_2\right)} \tag{6}$$

to obtain unknown parameters of the reference gear (u_1,v_1) and its mating gear (u_2,v_2,φ_1).

$$\left\{u_1(\varphi_1), v_1(\varphi_1), u_2(\varphi_1), v_2(\varphi_1), \varphi_2(\varphi_1)\right\} \in C^1 \tag{7}$$

The method is sufficiently general for any gear tooth surface geometry and for gear tooth surfaces that are out of alignment. For instance, Fig. 1 demonstrates the helical gear tooth surface contact in the case of in-plane misalignment.

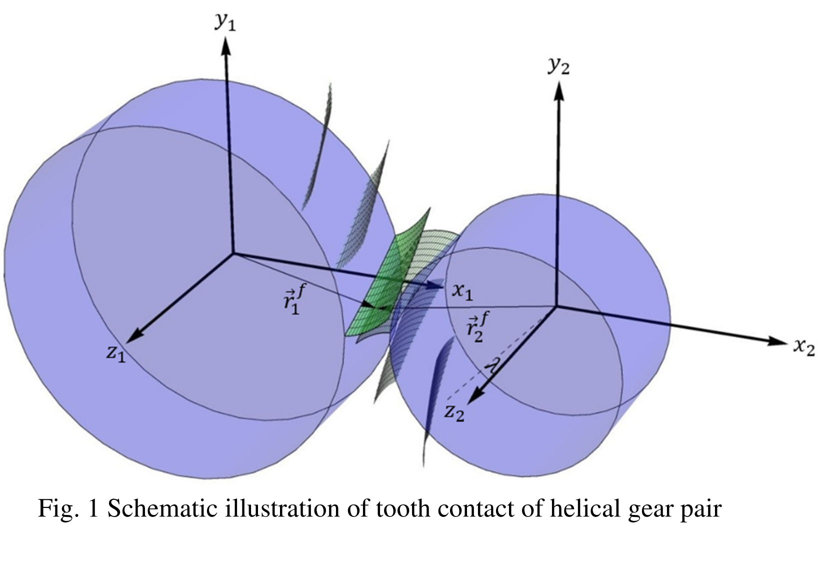


Fig. 1 Schematic illustration of tooth contact of helical gear pair

During the simulation procedure, the misalignment of the gear tooth can be analytically expressed through the rotation matrix $R(\lambda)$. The solution of the system is based on subroutine application with an iterative process. Such numerical solutions have a propensity to a heavily dependent on an appropriate selection of initial "guess values." The numerical algorithm may generate inaccurate results, such as convergent solutions relative to the real ones, or it may fail to converge at all if the guess values are not carefully chosen [9-10].

3. Model Computational Replication and Assessment

The conventional tooth contact equations will undergo rigorous analysis and simulations to evaluate the model's performance, identify its limitations, and assess its robustness for spur and helical gears. Additionally, the impact of varying initial conditions on the model's results will be thoroughly investigated.

3.1. Parametrization of spur gear tooth surface

The Wolfram Mathematica computing environment was used to simulate the benchmark solution [1]. In a single meshing position, the contact problem of two C^1 spur involute surfaces with comparable properties were taken into consideration. Table 1 displays the surface parameters.

Table 1 Parameters of the contacting surfaces

Parameters	Gear 1 and Gear 2
Number of teeth	25
Module (mm)	2.5
Pressure angle (degree)	20
Face width (mm)	25

As in a standard spur gear design, the rotation axes of both surfaces were chosen to be parallel to one another ($w_1 \| w_2$). The parameterization of the involute surfaces was determined and represented from a general equation in terms of specified parameters u_i and v_i for a specific machine tool setup. To localize the surface contact, the surfaces were adjusted by including additional variables in the involute surface equation. This ensured contact problem would have a single-point solution, in line with manufacturing procedures for profile modification (tip and root relief, crowning). The following are the involute's parametric equations:

$$x_i = r_b\left(\cos\theta_i + \theta_i \sin\theta_i\right) + \delta_x \tag{8}$$

$$y_i = r_b\left(\cos\theta_i - \theta_i \sin\theta_i\right) + \delta_y,\ i = 1, 2 \tag{9}$$

$$z_i = v_i \tag{10}$$

where r_b is a base radius. θ_i describes the involution angle:

$$\theta_i = \sqrt{\frac{u_i^2}{r_b^2} - 1} \tag{11}$$

The additional terms δ_x and δ_y refer to the tooth surface modification that has been applied. Furthermore, throughout the entire modeling process, only the longitudinal parabolic modification will be applied as follows:

$$\delta_y = a_p\left(v_i - \frac{b}{2}\right)^2 \tag{12}$$

where b is the face width of the gear tooth and a_p is the parabolic modification factor.

3.2. TCA outcomes for aligned spur gears

In the current simulation procedure, it is assumed that the teeth transfer without any axial or angular misalignment. A small amount of parabolic modification has been applied to the meshing gear tooth to achieve point contact, a_p = 0.0001 mm^{-1} and $\delta_x = 0$. As a result, the path contact will concentrate in the middle of the tooth surfaces. Fig. 2 illustrates the contact trajectory obtained using the established TCA model proposed by Litvin. To ascertain the position of the contacts, it is required to select a certain initial value to commence the iterative process to solve the nonlinear equations of gear coupling. To simplify calculations in the Wolfram Mathematica environment, the center of the surfaces was chosen as the starting values for the unknown parameters of the second gear tooth (u_2,v_2). For the unknown angle parameter (φ_2), a value of 0.03 rad was close to the correct solution of the first contact, and nearly 0.3 rad close to the last contact was selected.

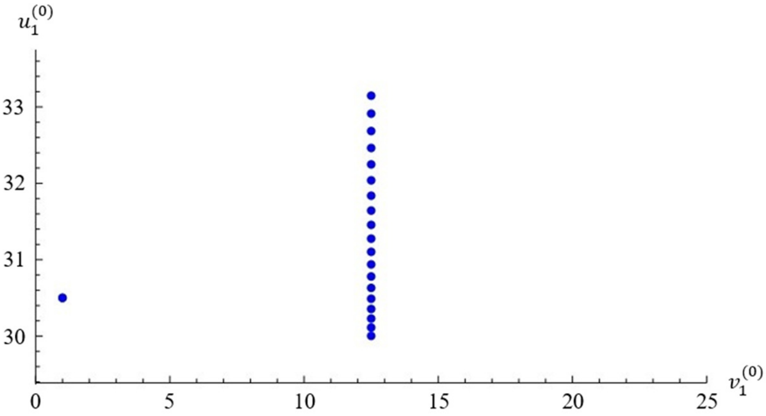
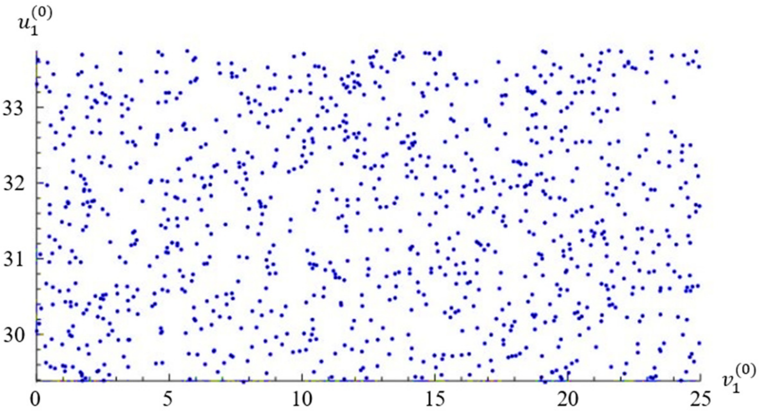


Fig. 2 The path of contacts on spur gear tooth surface

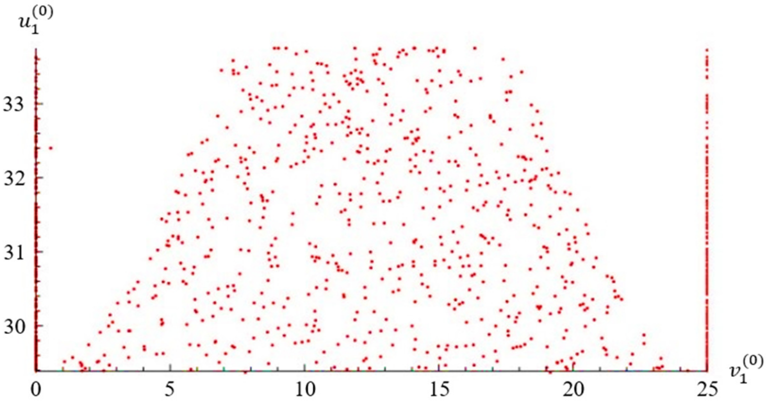
The final result was established after numerous simulations, repeated until divergence-related warning messages ceased. The outcome indicates model provides erroneous answers for some contact points despite careful initial value selection. By erroneous outcomes, points are meant where iterations move away from the required root or return to the starting point.

It was confirmed that the TCA algorithm is more sensitive to the angle position parameter φ_2 than other values such as u_1,v_1,u_2,v_2 [9-10]. Also, Litvin reported that to reach convergence, it is essential to first establish the angular location of the mating surface. Before using the TCA model, it is important to carry out a computationally intensive parametric sweep to address this problem. To establish the pitch point contact position in space, the domain of 1000 randomly generated starting values on the (u_1,v_1)-plane (first surface, S_1) was applied to the TCA method, Fig. 3(a). The outcomes of the solution (range) are provided in Fig. 3(b), where no single point converged. The midpoint of the second surface S_2 is chosen as the initial value for the $u_2^{(0)}$ and $v_2^{(0)}$ parameters during the 1000 simulation and $\varphi_2^{(0)}$ is set to zero.



(a) Domain

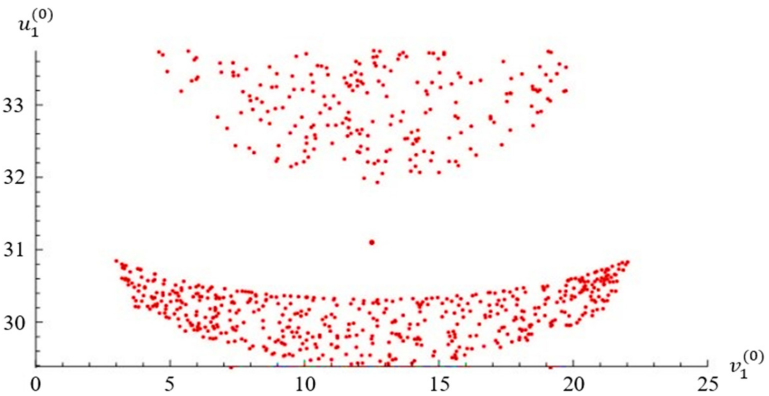
Fig. 3 Simulation results 1000 random points on tooth surface



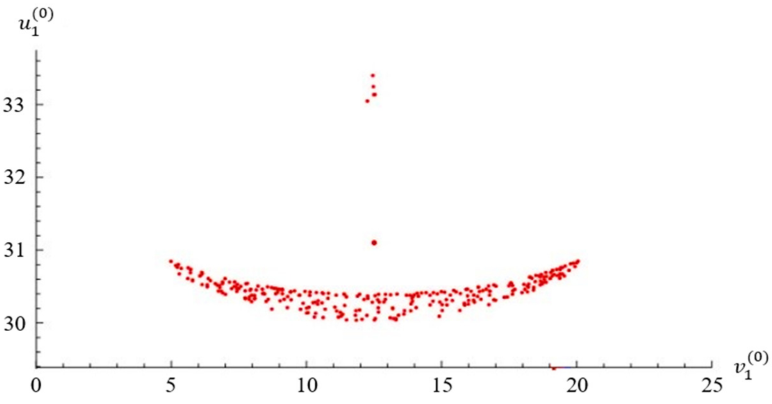
(b) Range

Fig. 3 Simulation results 1000 random points on tooth surface (continued)

The supplied data in Fig. 4 illustrates how effectively the established implicit TCA model converges to various starting values of the angle parameter $\varphi_2^{(0)}$. As was already indicated, the algorithm is quite dependent on the choice of $\varphi_2^{(0)}$, with values close to the right answer generating more convergence points. Only 480 out of 1000 points converge at an initial angle of $\varphi_2^{(0)}$ $= -1.1$ rad with a convergence rate of 48%, whereas results climbed to 670 points converge with an initial angle of $\varphi_2^{(0)} = -1.15$ rad. Finally, almost all points converged for $\varphi_2^{(0)} = -1.2$ rad, as the result of which pitch point contact position in space for both surfaces is: $u_1 =$ 31.103 mm; v_1 = 12.5 mm; $u_2 =$ 31.402 mm; v_2 = 12.5 mm; $\varphi_2 = -1.6$ rad at $\varphi_1 =$ 1.5707 rad. All of the aforementioned cases maintained the same conditions as the previous analysis, which employed the midpoints of the Σ_2 as the values for the parameters u_2^0 and v_2^0 respectively. The results obtained reveal the necessity of either performing a parametric sweep or using a large number of random beginning conditions to assure the convergence of the implicit conventional TCA model and prevent incorrect results, which in both situations is time-consuming and not computationally simple.

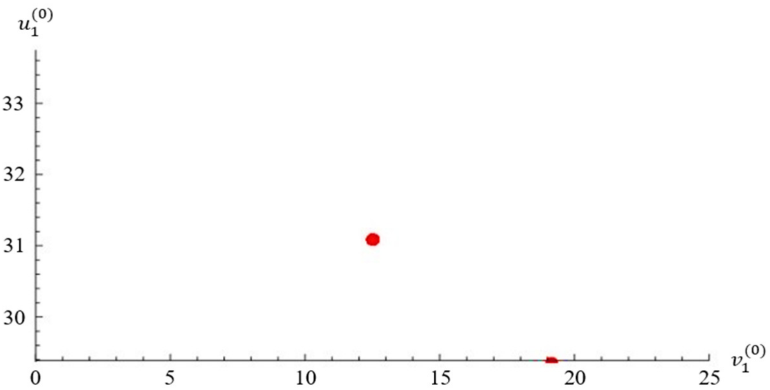


(a) $\varphi_2^{(0)} = -1.1$



(b) $\varphi_2^{(0)} = -1.15$

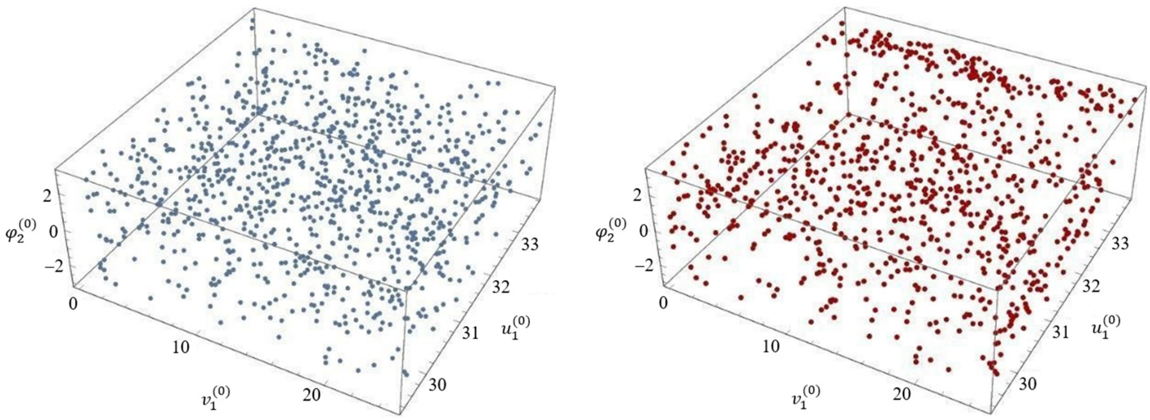
Fig. 4 Demonstration of converged and non-converged values on the guess cloud for Litvin's model



(c) $\varphi_2^{(0)} = -1.2$

Fig. 4 Demonstration of converged and non-converged values on the guess cloud for Litvin's model (continued)

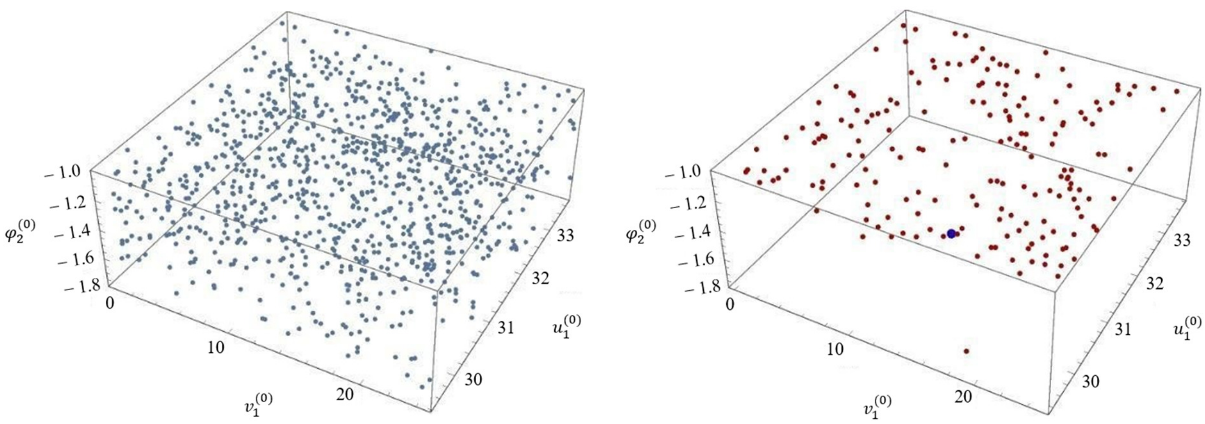
The domain of 1000 randomly generated starting values for ($u_1^{(0)}$,$v_1^{(0)}$,$\varphi_2^{(0)}$)-parameters applied to the TCA method, and was used to determine the pitch point contact position in space Fig 5(a). Fig. 5(b) shows the results of the solution (range), where all points failed to converge. During the 1000 simulation, the midpoint of the second is selected as the initial value for the $u_2^{(0)}$ and $v_2^{(0)}$ parameters.



(a) Starting guess values (b) Results of TCA solution

Fig. 5 Simulation results 1000 random points

To achieve a higher percentage of convergence compared to Fig. 5 for 1000 initial random values in ($u_1^{(0)}$,$v_1^{(0)}$,$\varphi_2^{(0)}$) space, the range of the selected random angular parameter $\varphi_2^{(0)}$ was narrowed to between -1.7 rad $< \varphi_2^{(0)} < -1$ rad, closer to the most converged answer $\varphi_2^{(0)} > -1.6$ rad from the previous simulation (Fig. 4). As a result, the simulation outcomes demonstrated a convergence of 818 points out of 1000, as shown in Fig. 6.



(a) Starting guess values (b) Results of TCA solution

Fig. 6 Outcomes of 1000 simulations of random starting points

It should be taken into consideration that the simplest contact model of a spur gear pair without any misalignments is being considered. However, to identify the contact position, further manipulations, such as reducing the range of selected starting values, are necessary. Additionally, there is an occasional need to visually assess the outcomes to ensure proper contact, as the model can produce inaccurate results in addition to divergent ones. As is known from iterative methods like Newton-Raphson, the possible causes of unsuccessful convergence may include the distant location of the initial guess from the root or the discontinuity of the considered function in the region where the root is being searched.

3.3. TCA outcomes for misaligned spur gears

To assess how the simulation results would change based on the presence of 0.1° in-plane misalignment, as in the previous section, the identical TCA process was carried out. The contact path computed using the validated TCA model proposed by Litvin is shown in Fig. 7. For the 2nd gear tooth's unknown parameters (u_2, v_2), the centers of the surfaces were chosen as the starting values, similar to the previous instance. However, for the unknown angle parameter $\varphi_2^{(0)}$, a value of 0.03 rad was close to the correct answer for the first contact, and nearly 0.3 rad close to the last contact was selected. The results indicate that, despite strong initial value selection support, the model produces incorrect answers at several contact points.

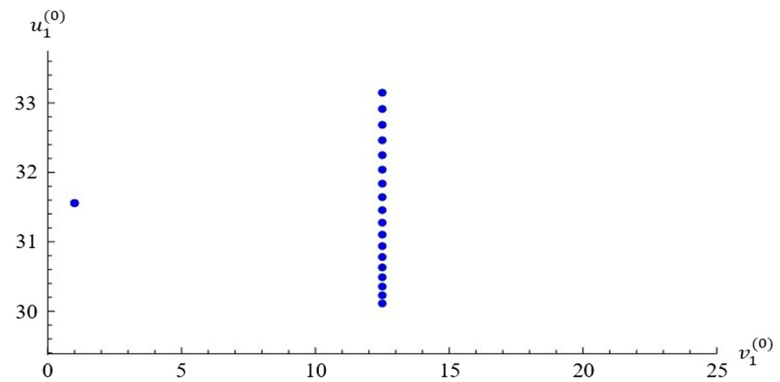
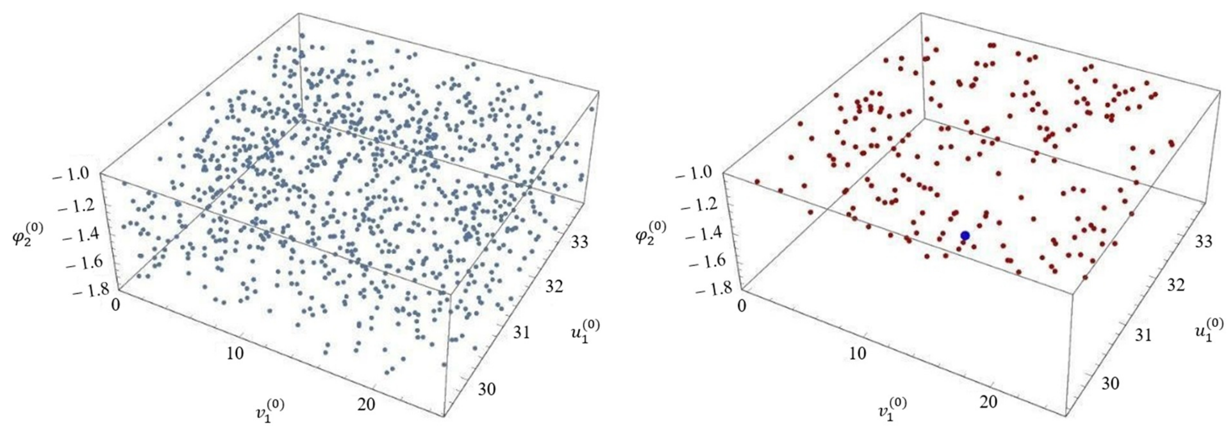


Fig. 7 The contact route along the in-plane misaligned spur gear tooth surface

The pitch point contact position for misaligned spur gear in space was calculated using the TCA technique and the domain of 1000 randomly generated starting values Fig. 8(a). The range of the chosen random angular parameter $\varphi_2^{(0)}$ was limited to -1.7 rad $< \varphi_2^{(0)} < -1$ rad to achieve a higher percentage of convergence. The simulation's results showed 80% convergence out of 1000 points, as seen in Fig. 8(b). In the following section, the simulation process performed by Litvin et al. [19] for helical gear engagement with out-of-plane misalignment will be repeated to verify the relevance of the traditional TCA model. This approach will help verify the model's effectiveness in predicting gear contact behavior under misaligned conditions and assess its applicability to real-world gear alignment challenges.



(a) Initial guess values (b) Results of the TCA model

Fig. 8 Results of 1000 simulations using random starting points

3.4. Actual TCA outcomes for helical gears with parallel axes

A general involute equation was applied to express the parameterization of the involute helical gear tooth surfaces in space in terms of the given parameters (u_i,v_i):

$$x_i = u_i \cos\left[\frac{r_b\left(\cos\theta_i + \theta_i \sin\theta_i\right)}{u_i} + \frac{v_i \sin\beta}{r_b}\right] + \delta_x \tag{13}$$

$$y_i = u_i \sin\left[\frac{r_b\left(\cos\theta_i + \theta_i \sin\theta_i\right)}{u_i} + \frac{v_i \sin\beta}{r_b}\right] + \delta_y \tag{14}$$

$$z_i = v_i \tag{15}$$

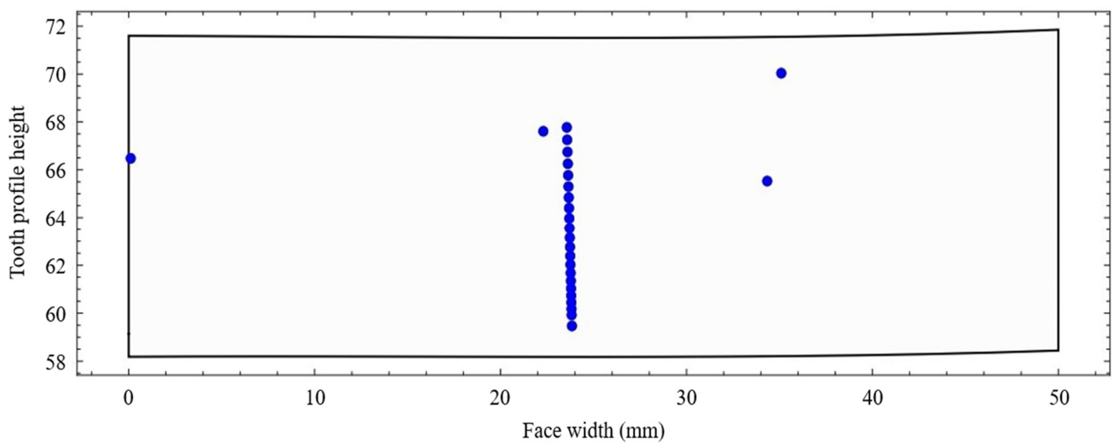
where β is a helix angle, θ_i is the involution angle, and $\delta_{x,y}$ is tooth surface modification function.

In this section, the modeling process performed by Litvin et al. [19] for both aligned and misaligned helical gear meshing is reproduced. For an objective assessment of the TCA model, the same parameters were selected for pairs of helical gears and the size of the crown with misalignments, as shown in Table 2.

Table 2 Parameters of the contacting surfaces

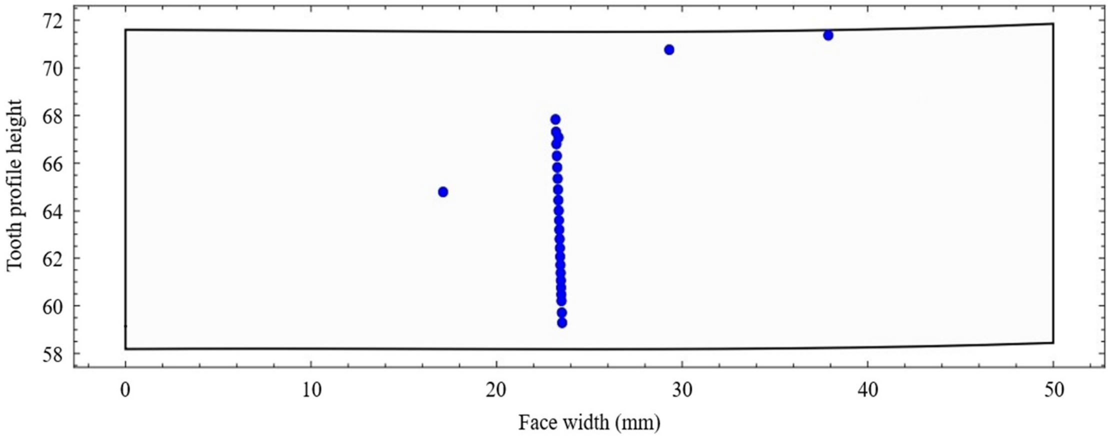
Parameters	Gear 1	Gear 2
Number of teeth	25	77
Module (mm)	5	
Pressure angle (degree)	27.5	
Helix angle (degree)	20	
Face width (mm)	40	
Cross angle (degree)	0	
Parabola parameter of profile crowning	1.4×10^{-3}	8×10^{-5}

The system of nonlinear equations in Eqs. (1)-(7) numerical solution can be used to determine the contact path. The success of this numerical solution, however, heavily hinges on the choice of proper initial “guess values” due to the intricacy of the contact equations. If the starting values are not chosen properly, the numerical algorithm may result in an inaccurate result with flawed contacts or inadequately converged solutions, as seen in Figs. 9-10. Points that produce incorrect results are those whose iterations either return to the initial point or move the approximation farther from the required root. Fig. 9(a) demonstrates the result of aligned helical teeth engagement, and Fig. 9(b) contacts in the presence of an alignment error $\Delta\lambda$ = −3 arcmin. In both cases the $\varphi_2^{(0)}$ close to the correct solution for 0.05 rad, but with randomly chosen starting values for $u_2^{(0)}$ and $v_2^{(0)}$ parameters.



(a) Aligned contact

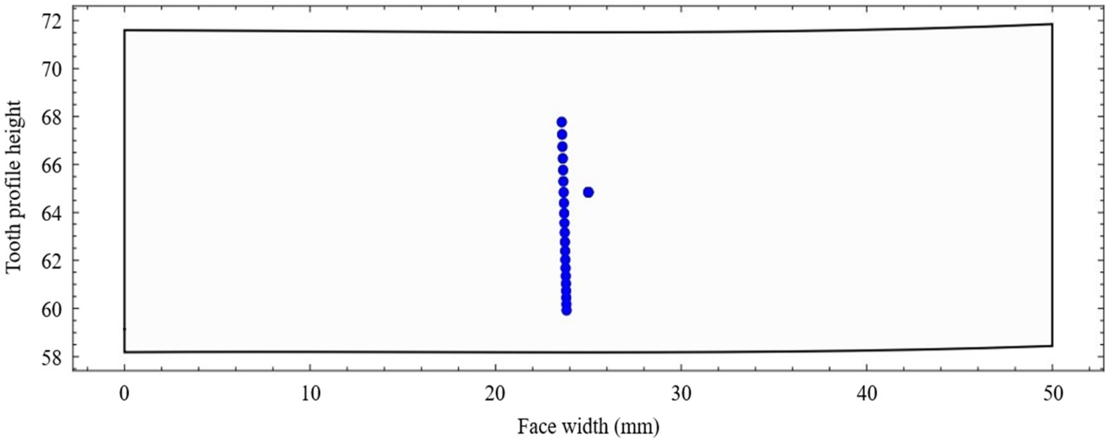
Fig. 9 Demonstration of path contact



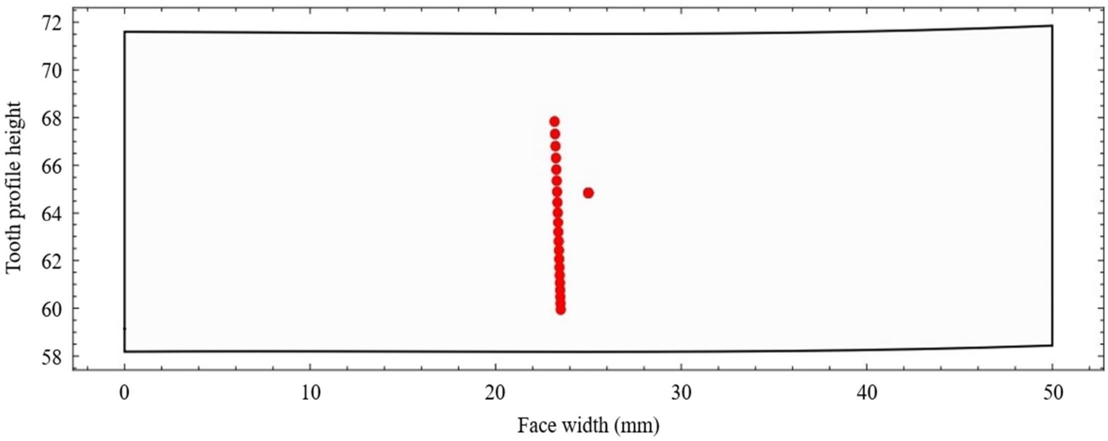
(b) Misaligned contact

Fig. 9 Demonstration of path contact (continued)

Fig. 10 shows the same cases of path contact but now $\varphi_2^{(0)}$ is close to the correct solution for 0.05 rad and the starting value is close to the actual contact for $u_2^{(0)} - u_2^{sol}$ = 4.89 mm and $v_2^{(0)} - v_2^{sol}$ = 1.44 mm. From Figs. 9-10, it can be concluded that even with reasonable assumptions about the initial values and without human observation, the results cannot always be fully trusted.



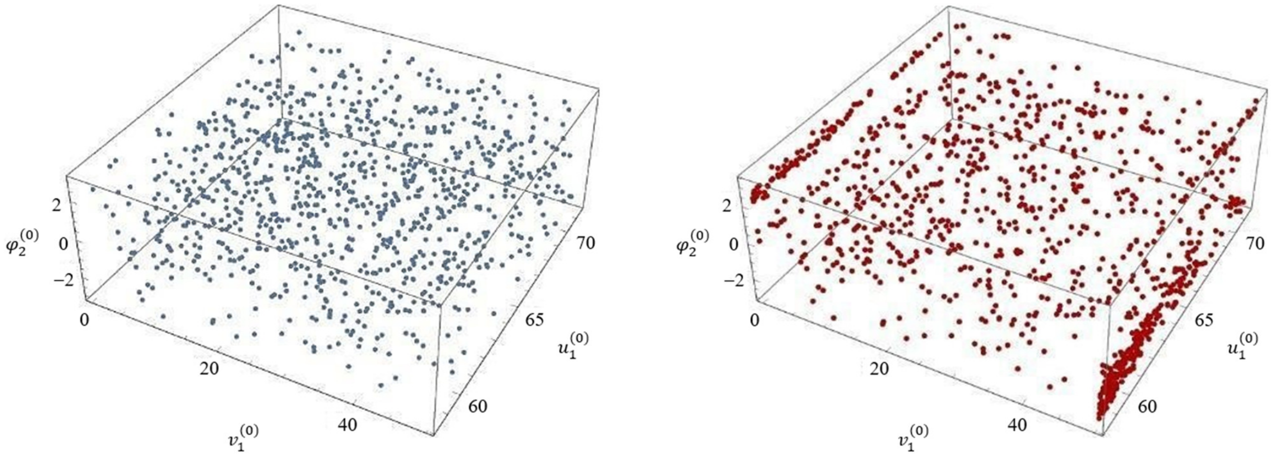
(a) Aligned contact



(b) In-plane misaligned contact

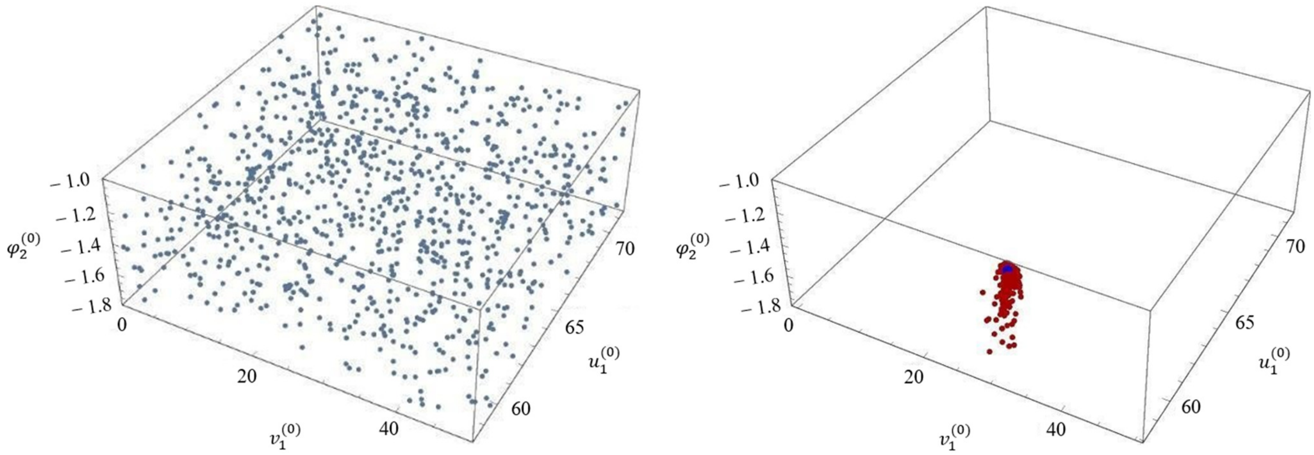
Fig. 10 The path contact of helical gears

The domain of 1000 randomly generated starting values for the ($u_1^{(0)}$,$v_1^{(0)}$,$\varphi_2^{(0)}$)-parameters were used to calculate the pitch point contact position of parallel axes helical gears with $\Delta\lambda = -3$ arcmin in-plane misalignment in space, as shown in Fig. 11(a). The results of the solution (range) are shown in Fig. 11(b), where all points failed to converge.



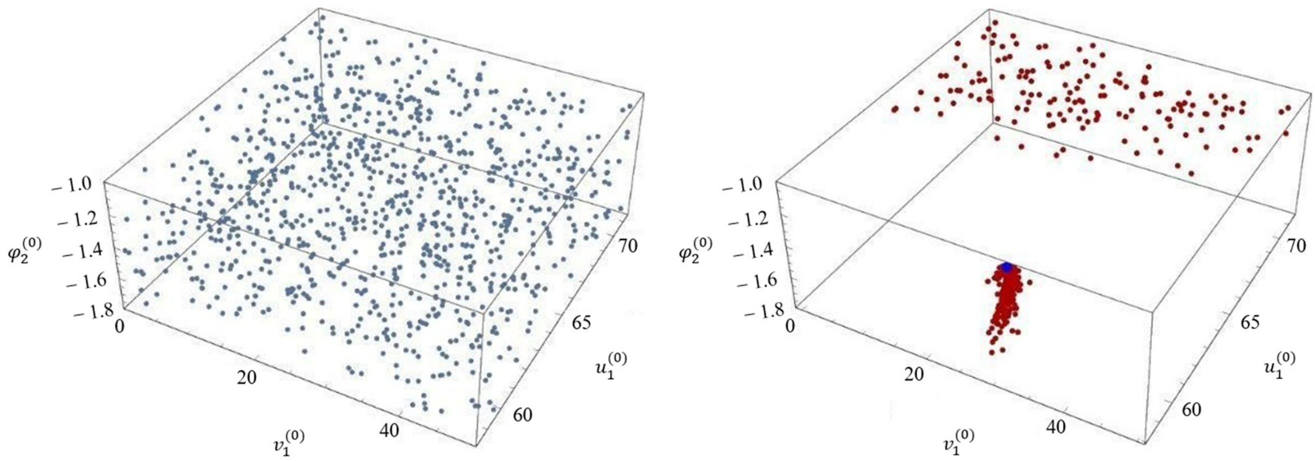
(a) Starting guess values (b) Litvin's TCA solution

Fig. 11 Simulation results of 1000 random points on helical gear tooth surface



(a) Initial guess values (b) Conventional TCA solution

Fig. 12 Results of 1000 random points simulation on the surface of aligned helical gears



(a) Starting guess values (b) Results of conventional TCA solution

Fig. 13 Simulation results of 1000 random points on misaligned helical gear tooth surface

Figs. 12-13 depict the same simulation procedure as Fig. 11, for aligned and misaligned engagement, but with reduced diapason of the selected random angular parameter $\varphi_2^{(0)}$ so that they were generated between -1.7 rad $< \varphi_2^{(0)} < -1.3$ rad, where the upper limits were close to solution by $-1.3 - \varphi_2^{sol} = -0.335$ rad and the lower limits by $-1.7 - \varphi_2^{sol} = -0.065$ rad and actual solution is $\varphi_2^{sol} = -1.635$ rad. In these cases, the proportion of convergences was better than in Fig. 11, but they were still only 13-15%. Also, from Figs. 12(b)-13(b) it is notable that some points approached around the correct solution, but did not reach it. As previously indicated, a system of nonlinear equations based on the traditional TCA model can result in several converged solutions, however, some of them will not converge to the actual point of contact as expected. Afterward, the number of Newton-Raphson iterations was increased to determine whether the failed converged points would eventually reach the precise root. The analysis showed that the results remained consistent, demonstrating that this model can produce incorrect results.

4. Conclusions

This study provided a review of the mathematical framework of the conventional TCA model introduced by Litvin, identifying key assumptions and computational challenges as described in Section 2. The established methodology was implemented using the Wolfram Mathematica computing system to evaluate its performance and accuracy in iterative numerical calculations. The focus was on contact analysis between the spur and helical involute gear surfaces, as well as the stability and accuracy of the numerical solutions obtained. Section 3 involved the replication and assessment of the TCA model for both aligned and misaligned gear configurations, analyzing its sensitivity to initial guess values, as well as overall performance. The findings underscore the model's limitations in producing reliable results without requiring significant manual adjustments and computational resources.

(1) It was demonstrated that the conventional TCA approach is highly sensitive to the initial guess, particularly the starting value of the angular position. Due to the complexity of Litvin's non-linear equations and multiple roots (unknown parameters), convergence problems are often encountered. When initial guesses are far from the actual solutions, iterations may diverge or oscillate without converging on the correct root, leading to infinite looping. The method is not inherently stable.

(2) The automatic guess value determination technique suggested by Litvin aims to find suitable "guess values" for convergence but results in an impractical and computationally expensive implementation. Repeating the modeling procedure described in previous studies led to inconsistent results with random loss of convergence. Therefore, the method thus requires human supervision, which contradicts the purpose of the algorithm.

(3) This analysis establishes that the conventional TCA method suffers from inconsistency and convergence issues, necessitating further development to ensure robustness and reliability in numerical root-finding techniques. Improvements are required to avoid time-consuming repeated simulations in cases of divergence.

Funding

This work was supported by the Zhas Galym project for 2024-2026 from the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant No. AP22683443).

Conflicts of Interest

The authors declare no conflict of interest.

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