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A Direct Lyapunov-Backstepping Approach for Stabilizing Gantry Systems with Flexible Cable

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Abstract

Trolley positioning and payload swinging control problem of a flexible cable gantry crane system are addressed in this paper. The system's equations of motion that couple the crane's cable and actuators dynamics are derived via extended Hamilton's principle. The control signal is designed based on the Lyapunov direct method to derive control force and backstepping technique is employed to determine input signal for the actuator. The stability of the closed loop system is proven analytically. Numerical simulations are included to demonstrate the effectiveness and robustness of the closed-loop system.

Keywords: flexible systems, overhead crane, field oriented control, Lyapunov direct method

1. Introduction

Nowadays, gantry crane systems are widely used in industrial and logistic applications because of their flexibility in load handling. However, swinging payload phenomenon causes slowing down goods handling operations and can be a potential threat to human and surrounding devices. Certain types of payloads can ignite multi-modes or double-link pendulum effects [1-4]. In addition, characterized as a class of under-actuated systems, precisely controlling trolley position and suppressing payload vibration simultaneously pose many challenges for control engineers.

In order to overcome the aforementioned control problem, various approaches are considered. A conventional robust linear control law is proposed to control the overhead crane [5]. Since crane system is a nonlinear coupling system, instead of linear control, many researchers focus on nonlinear control approaches. A decoupling control law is proposed to asymptotically stabilize trolley position and swing angle of the payload [6]. However, the designed control only guarantees bounded swing angle. An improvement [7] is made with varying crane rope length that is meaningful in practice is considered. A switching control action is derived based on feedback linearization technique. Position control and vibration suppression of gantry crane with coupling effect between trolley and payload motions are taken into account [8]. However, the obtained results are relatively limited in practice because of system's parameters and actuator's dynamics variations. In order to deal with system uncertainty, an adaptive mechanism is integrated in proposed control law suggested [9]. Well-known with its robustness against system uncertainties and disturbances, sliding mode control is applied in gantry control [10]. However, it is need to cooperate with a pre-shape input to gain better performances [11]. Several adaptive schemes [12-14] for gantry control also presented. Intelligent control schemes are also considered to control the crane system such as fuzzy control [15] or neural network [16].

Instead of feedback controls, some other researches consider feed-forward control approaches where control actions from operators are modified before sending to the gantry actuators as shown [17-18]. The advantage of pre-shape input technique

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over feedback control is that measurement of system states is not required but a full knowledge of the system must be available. To rectify this drawback, pre-shape input method can be hybridized with a robust control as indicated [19-20].

The limitation of aforementioned studies is an assumption of pendulum motions for the payload. The assumption results in a system of ordinary differential equations govern system motions. However, practical applications have shown that it is not the case, and gantry cable actually considered as a flexible system whose motions are modeled as a system of partial differential equations. Boundary controllers for stabilizing the flexible rope crane system based on Lyapunov's direct approach are developed [21-23]. The flexible rope where coupled longitudinal-transverse, transverse-transverse motions and 3D model are investigated [24-27]. Dynamics of the rope without model truncation is investigated, however the dynamics of the actuator are totally ignored. The ignorance of the actuator dynamics might to system instability.

This paper directly designs a gantry control in consideration with flexible rope but in other direction. We construct a distributed model of the overhead crane in which the mass and the flexible of payload suspending cable are fully taken into account. The analytical mechanics including Hamilton's principle is used to construct the crane model. Based on the obtained model, the controller is then designed systematically with the help of backstepping control.

The rest of the paper is organized as follows. The mathematical model of flexible overhead crane system is presented in Section 2. In section 3, control design is developed based on the Lyapunov direct method and backstepping technique. The numerical simulation is performed in Section 4 to show the efficiency of the proposed control design. Conclusions and further studies are presented in Section 5.

2. Mathematical Model

An overhead crane system is illustrated in Fig. 1, where

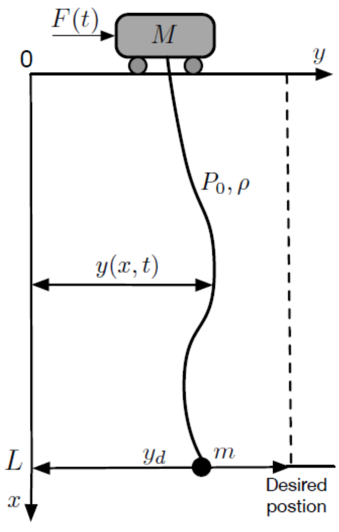

Fig. 1 A gantry crane system

$y(x,t)$ and y_d are the transverse motions of the crane's cable and the target position of the payload, respectively. P_0 is the cable tension, ρ is the mass per unit length of the cable, and L is the length of the cable. M and m are trolley's and payload's masses, respectively. The force $F(t)$ is generally generated by an induction motor. The kinetic energy of the system is given as

$$T = \frac{M}{2}(\frac{\partial y\ 0}{\partial t})^2 + \frac{\rho}{2}\int_0^L (\frac{\partial y}{\partial t})^2 dx + \frac{m}{2}(\frac{\partial y\ L}{\partial t})^2 \tag{1}$$

In Eq. (1) and from now onward the argument (x,t) is omitted for neat representation. In addition, $y(0)$ and $y(L)$ are used to denote $y(0,t)$ and $y(L,t)$, respectively. The potential energy can be expressed as

$$P=\frac{P_0}{2}\int_0^L(\frac{\partial y}{\partial x})^2dx \tag{2}$$

where P_0 is the tension of the cable. The work done by the external control force is given as

$$\mathrm{W}=F(t)y(L) \tag{3}$$

Remark 1. Bending stiffness of the cable is considerably small so that potential energy due to bending stiffness can be ignored. The cable is assumed to be inextensible, and the cable deforms in Oxy plane.

The extended Hamilton's principle is expressed as follows

$$\int_{t_1}^{t_2}\delta(T-P+\mathrm{W})dx \tag{4}$$

Substituting Eqs. (1), (2) and (3) into Eq. (9) results in

$$\int_{t_1}^{t_2}\int_0^L\left[\left(-\rho\frac{\partial y^2}{\partial t^t}+P_0\frac{\partial^2 y}{\partial x^2}\right)\delta dx-P_0\frac{\partial y}{\partial x}\Big|_0^L+M\frac{\partial y^2(0)}{\partial t^2}\delta y(0)dt+m\frac{\partial y^2(0)}{\partial t^2}\delta y(0)+F(t)\delta y(0)\ dt=0 \tag{5}$$

Using integration by parts, the equations of motions and boundary conditions of the crane system can be given as

$$\rho y_{tt}-P_0y_{xx}=0 \tag{6}$$

$$My_{tt}(0)+P_0y_{\mathrm{x}}(0)=F(t) \tag{7}$$

$$my_{\mathrm{tt}}(L)-P_0y_{\mathrm{x}}(L)=0 \tag{8}$$

The force acting on the trolley $F(\mathrm{t})$ can be calculated in term of the motor torque as

$$F(\mathrm{t})=\frac{i\eta}{R_b}m_M \tag{9}$$

where R_b is the radius of the drum, i is the transmission ratio of the gearbox and η is the efficiency of the transmission system. Mathematical model of the asynchronous motor can be written as follows:

$$\dot{i}_{sd}=-\left(\frac{1}{\sigma}+\frac{1-\sigma}{\sigma T_r}\right)i_{sd}+\omega_s i_{sq}+\frac{1-\sigma}{\sigma T_r}\psi'_{rd}+\frac{1-\sigma}{\sigma}\omega\psi'_{rd}+\frac{1}{\sigma L_s}u_{sd} \tag{10}$$

$$\dot{i}_{sd}=-\omega_s i_{sd}-\left(\frac{1}{\sigma T_s}+\frac{1-\sigma}{\sigma T_r}\right)i_{sd}-\frac{1-\sigma}{\sigma}\omega\psi'_{rd}+\frac{1-\sigma}{\sigma T_r}\psi'_{rd}+\frac{1}{\sigma L_s}u_{sq} \tag{11}$$

$$\dot{\psi}'_{rd}=\frac{1}{T_r}i_{sd}-\frac{1}{T_r}\psi'_{rd}+\ \omega_s-\omega\ \psi'_{rd} \tag{12}$$

$$\dot{\psi}'_{rq}=\frac{1}{T_r}i_{sq}-\ \omega_s-\omega\ \psi'_{rd}-\frac{1}{T_r}\psi'_{rq} \tag{13}$$

$$m_M=\frac{3}{2}K_m z_p\psi_{rd}i_{sq} \tag{14}$$

where i_{sd} and i_{sq} are direct and quadrature components of stator current. T_r and T_s are rotor and stator time constants, z_p is number of pole pairs, and σ is total magnetic leakage factor. $K_m=\frac{L_m}{L_r}$, where L_m and L_r are mutual and rotor inductance.

$\psi'_{rd} = \dfrac{\psi_{rd}}{L_m}$ and $\psi'_{rq} = \dfrac{\psi_{rq}}{L_m}$, where ψ_{rd} and ψ_{rq} are dq components of the rotor flux. The coupled electrical-mechanical systemis rewritten as follow:

$$\rho y_{tt} - P_0 y_{xx} = 0 \tag{15}$$

$$M y_{tt}(0) - \mathrm{P}_0 \mathrm{y}_{\mathrm{x}}(0) = \Omega i_{sq} \tag{16}$$

$$\dot{i}_{sq} = -\theta_1 i_{sd} - \theta_2 i_{sq} - \theta_3 + \theta_4 u_{sq} \tag{17}$$

$$m y_{tt}(\mathrm{L}) + \mathrm{P}_0 \mathrm{y}_{\mathrm{x}}(\mathrm{L}) = 0 \tag{18}$$

where

$$\theta_1 = \omega_s,\ \theta_2 = \frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r},\ \theta_3 = \frac{1-\sigma}{\sigma}\omega\psi'_{rd} \tag{19}$$

and

$$\theta_4 = \frac{1}{\sigma L_s},\ \Omega = \frac{3}{2}\frac{i\eta}{R_b} K_m z_p \psi'_{rd} \tag{20}$$

Remark 2. Eqs. (15)-(18) is derived under a condition that the rotor flux orientation is obtained, i.e., $\psi'_{rq} = 0$. Moreover, it is assumed that i_{sd} and ψ'_{rd} are kept constants by current and flux controllers and their values are available for feedback. In addition, the current controller has the ability of decoupling i_{sd} and i_{sq} .

3. Control Design

The control objective is to simultaneously stabilize the trolley and the payload at the desired position. An investigation of the system given in Eqs. (15)-(18) shows that the system is of strict-feedback form. Hence, in this paper, backstepping technique will be employed to design the control input.u_{sq} The choice of back-stepping as a design tools make it ready if system parameters adaptation is needed. The control design process comprises of two steps. In order to satisfy the control objective, at first we take i_{sq} as a control and define.

$$z = \Omega i_{sq} - \alpha \tag{21}$$

where is *a* virtual control. Consider the following Lyapunov candidate function

$$\mathrm{W} = \frac{\rho}{2}\int_0^L y_t^2 dx + \frac{P_0}{2}\int_0^L y_x^2 dx + \frac{m}{2} y_t^2(\mathrm{L}) + \frac{M}{2} y_t^2(0) + \frac{\Delta}{2}[y(0) - \mathrm{y_d}] \tag{22}$$

where D is a strictly positive constant. It is straightforward to show that V can be lower and upper bounded as below

$$\mathrm{W} \geq \gamma_1 [\int_0^L y_t^2 dx + \int_0^L y_x^2 dx + y_t^2(\mathrm{L}) + \mathrm{y}_t^2(0) + [y(0) - y_d]^2 \tag{23}$$

and

$$\mathrm{W} \leq \gamma_2 [\int_0^L y_t^2 dx + \int_0^L y_x^2 dx + y_t^2(\mathrm{L}) + \mathrm{y}_t^2(0) + [y(0) - y_d]^2 \tag{24}$$

where

$$\gamma_1 = \frac{1}{2}\min(\int_0^L y_t^2 dx, \int_0^L y_x^2 dx,\ y_t^2(\mathrm{L}),\ y_t^2(0),\ [y(0) - y_d]^2) \tag{25}$$

and

$$\gamma_2 = \frac{1}{2}\min(\int_0^L y_t^2 dx, \int_0^L y_x^2 dx,\ y_t^2(\mathrm{L}),\ y_t^2(0),\ [y(0) - y_d]^2) \tag{26}$$

Taking time derivative with respect to Eq.(26),then

$$\begin{aligned}\dot{\mathrm{W}} &= \mathrm{P}_0 y_t y_x \Big|_0^L - P_0 y_t(\mathrm{L}) y_x(\mathrm{L}) + y_t(0)[\alpha_1 + z_1 + P_0 y_x(0)] + \Delta(y(0) - y_d) y_t(0) \\ &= y_t(0)\{\alpha + z + \Delta[y(0) - y_d]\}\end{aligned} \tag{27}$$

Eq. (27) suggests that virtual control a_1 can be chosen as follows

$$\alpha = -k y_t(0) - \Delta[y(0) - y_d] \tag{28}$$

where k is a strictly positive constant. In the second step, the actual control input u_{sq} is designed to regulate z_1 at the origin. To archive this target, we consider a Lyapunov candidate function as follows

$$V = \mathrm{W} + \frac{1}{2} z^2 \tag{29}$$

Taking time derivative with respect to Eq. (29), it yields

$$\dot{V} = -k_1 y_t^2(0) + z[-\frac{\theta 1}{\Omega 1} i_{sd} - \frac{\theta 2}{\Omega} i_{sq} - \frac{\theta 3}{\Omega} + \frac{\theta 4}{\Omega} u_{sq} - k_1 y_{tt}(0) - (\Delta - 1) y_t(0)] \tag{30}$$

The actual control input u_{sq} can be derived as

$$\frac{\theta_4}{\Omega} u_{sq} = \frac{\theta_1}{\Omega_1} i_{sd} + \frac{\theta_2}{\Omega} i_{sq} + \frac{\theta_3}{\Omega} + k_1 y_{tt}(0) + (\Delta - 1) y_t(0) \tag{31}$$

where k_2 is positive constant.

Remark 3: The control input u_{sd} can be derived based on backstepping control method as follows

$$\frac{1}{s} u_{sd} = \frac{1}{T\sigma} i_{sd} - \omega_s i_{sq} - \frac{1-\sigma}{r} \hat{\psi}'_{rd} + (\frac{1}{T_r} - c_1)(i_{sd} - \hat{\psi}'_{rd}) + c_1 T_r \frac{d\hat{\psi}'_{rd}}{dt} + T_r \frac{d^2 \hat{\psi}'_{rd}}{dt^2} - c_2 z_2 - \frac{1}{T_r} z_1 - d_2 z_2 \theta \tag{32}$$

where

$$\theta_2 = (\frac{1-\sigma}{\sigma T_r})^2 + (\frac{1-\sigma}{\sigma}\omega)^2 \tag{33}$$

c_1 and c_2 are strictly positive constant, z_1 and z_2 are errors between desired and virtual control when designing the flux controller.

The control design is completed and it is straightforward to show that with the selected control input u_{sq} render the first time derivative of the Lyapunov candidate function $\dot{V}$ as

$$\dot{V} = -k_1 y_t(0) - k_2 z \le 0 \tag{34}$$

Eq. (34) shows that $V(\mathrm{t})$ is upper bounded by $V(0)$. This consequently implies that $y_t(L)$, $y_t(0)$ and $y_t(0) - y_d$ are bounded. Further investigation of Eq. (34) can prove exponential convergence to y_d of $y(0)$ and $y(L)$.

4. Numerical Simulations

In order to verify the effectiveness of the proposed feedback control. Simulations are carried out using an induction motor of 7.5kW (other motor power ranges can be applied with no loss of generality). The closed loop system is simulated in

Matlab/Simulink environment. Simulation parameters are given in Table 1.

Table 1 Induction motor parameters

Parameter	Value
Nominal power	PN = 2,5kW
Pairs of pole	Pp = 2
Nominal current	UN = 340V
Nominal speed	n = 1400 rpm
Stator resistance	RS = 2.521Ω
Stator inductance	LS= 0.1825 H
Rotor resistance	RS = 0.976
Rotor inductance	LR = 0.1858 H
Inertial moment	J = 0,117 kg.m2

The simulation is carried out with the mass of the trolley is of 100kg, payload mass and cable length are of 400kg and 5m, respectively. Simulation scenario is to regulate trolley and payload position to a desired position of 5m from the initial condition. It is assumed that initially the positions of the trolley and payload are coinciding.

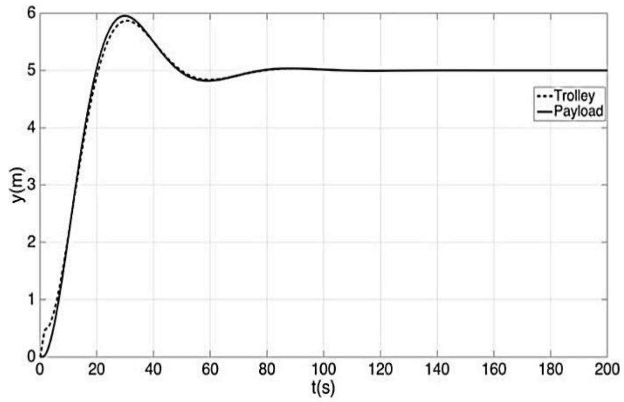

Fig. 2 System response of the trolley and payload without control

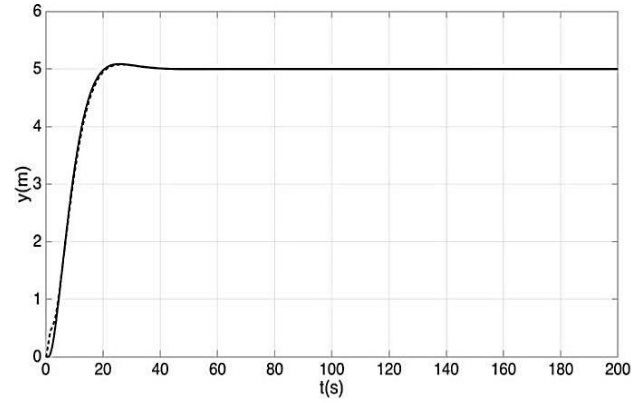

Fig. 3 System response of the trolley and payload with control

Fig. 2 clearly represents large fluctuation of the trolley and the payload. This phenomenon is undesirable in practice.

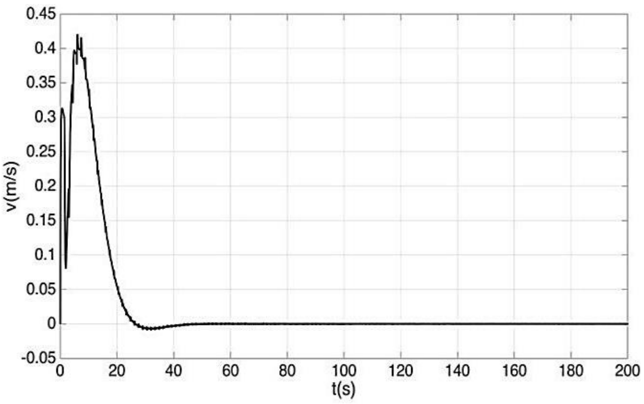

Fig. 4 Velocity response of the trolley with control

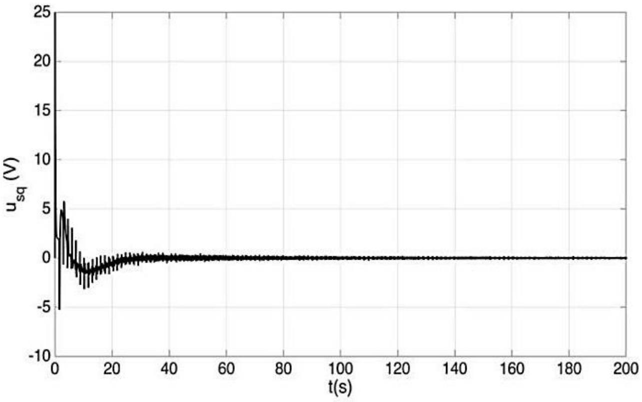

Fig. 5 Control input

It can be seen from the simulation results the effect of control action to the system. Without control, the payload variation with the maximum value of approximate 1m around the desired position. When the control is activated, the swing angle of the payload is reduced considerably. Finally, the payload mass and cable length are assigned to 600kg and 7m, respectively. Due to heavier load and longer cable, higher payload fluctuation is expected in comparison with the previous results.

Numerical simulation indicates that the effectiveness of the proposed control design. Trolley position is regulated at the desired value after 20s, and the payload also tracks the target after a few oscillations. In addition, the control input is of the applicable practice range.

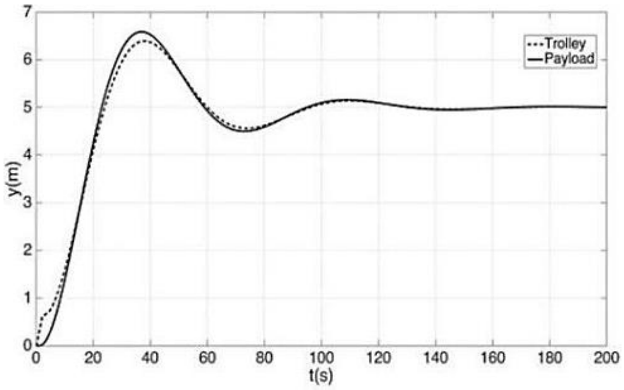

Fig. 6 System response of the trolley and payload without control

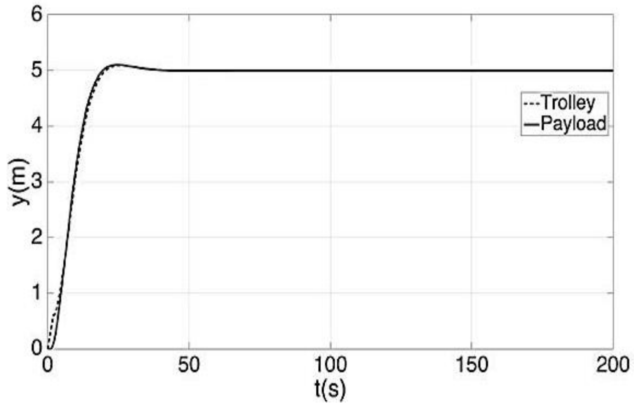

Fig. 7 System response of the trolley and payload with control

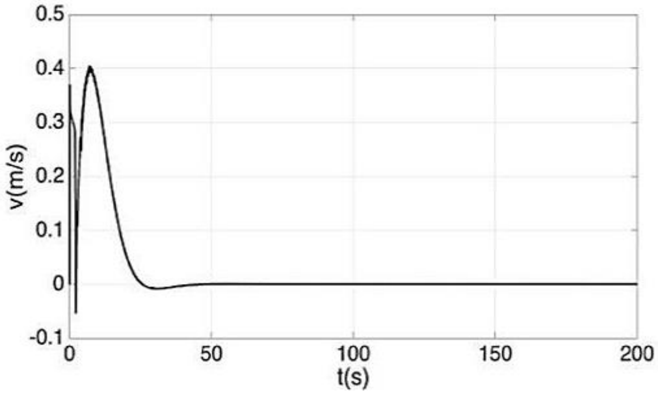

Fig. 8 Velocity response of the trolley with control

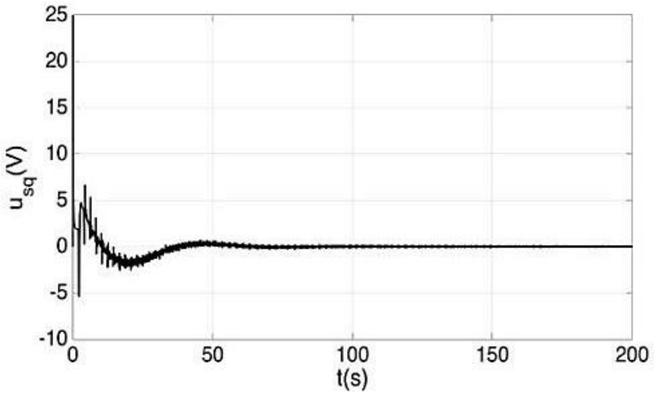

Fig. 9 Control input

5. Conclusion

A design of a position and vibration suppression control of a gantry crane system is demonstrated in this paper. Based on energy approach, a system of partial differential and ordinary differential equations that govern the system's motions including cable and actuator dynamics are derived. The system dynamics represent flexibility of the gantry cable. The Lyapunov direct method and backstepping technique are employed to design the controller. Stability and the effectiveness of the closed-loop system are verified analytically and illustrated numerically. The direct extension of the paper is to consider the motion of the system in three-dimensional space.

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